Studies on the potential of a reprocessing camapign of the GOCE observations inline with the time-wise method

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Abstract

This technical note tries to summarize studies which were performed within GOCE HPF WP6000 to detect remaining potential in the GOCE mission data set. Based on studies which were already performed by team members and and new upcoming ideas, this document tries to collect the results of the studies and ideas to propose a reprocessing of the entire mission data set. The focus is on the remaining potential for the final product, a static global gravity field model derived purely based on GOCE observations following the basic ideas and the philosophy of the so called time-wise approach.

In addition, some thoughts on further products are collected, which could be provided by WP6000. These products are meant to simplify the usage of the product (e.g. the covariance matrix), to improve availability, or with modifications of different aspects (models for different time periods, including different observation types).

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1 Introduction

Since the release of the fifth generation of global gravity field models from the GOCE mission in July 2014 (Brockmann et al., 2014), some effort was spend by WP6000 to study whether there is remaining potential in the original GOCE observations used for global gravity field determination. The goal is to find aspects in the data, the used algorithms and in the used models which might lead to improved global gravity field products derived from the GOCE mission. Based on the usage of the models in different applications, models produced by groups outside the HPF and some detailed data and model analysis some points came up, which might be reconsidered in order to produce more precise and thus better global gravity field models. This document is used to collect the studies and ideas which are suited for further studies. It tries to quantify expected improvements of the model and proposes a strategy for a reprocessing campaign. Furthermore, some additional products are proposed, allowing for a more flexible usage of the models and products.

2 Studies and strategies to improve the SST-hl processing

Kinematic orbit positions have been used within WP6000 to determine the long wavelength part of the gravity field. This has been an integral part of the time-wise method since the first proposal for the mission. The official precise science orbits have been generated at the Astronomical Institute at the University of Bern (Bock et al., 2011). The kinematic orbit product formed the basis for the SST-hl processing for the time-wise approach. Initially the energy balance approach was applied to estimate gravity field parameters from kinematic orbit positions. This was done for release 1 to 3. For the fourth time-wise solution the energy balance approach was replaced by the short arc integral approach, due to the fact that it outperforms the energy balance approach by a factor of $\sqrt{3}$ (Ditmar and Sluijs, 2004, Baur et al., 2014). Finally for release 5 an orbit computed at the Institute of Geodesy of the TU Graz has been used. The orbit was computed using a newly developed software applying the raw observation approach. The switch was made, because investigations revealed that better results could be achieved when using the IfG orbit as input opposed to the AIUB orbit. Especially an error along the geomagnetic equator had a strong influence on the quality of the SST-hl solution. The subsequent sections shortly describe the raw observation approach, some special aspects in the IfG orbit processing as well as mission specific approaches to reduce observed errors along the magnetic equator. Some of these approaches have been already used to generate the kinematic orbit for the SST-hl part of EGM_TIM_RL05, whereas others still provide some room for improvements.

Although the quality of the kinematic orbit positions is the main contributor to the achievable accuracy of a gravity field solution based on the SST-hl concept, it is also important to apply state-of-the-art methods for gravity field retrieval. For release 4 and 5 the short arc integral approach has been used. Investigations have shown that available methods, like the acceleration approach, the celestial mechanics approach, or the short arc integral approach provide equal results (Baur et al., 2014). Nevertheless, further investigations shall be conducted to identify possible improvements, for example in terms of error modeling or outlier handling. Continuous quality monitoring will ensure a consistent reprocessing of the complete GOCE data set and thus lead to an improved final gravity field solution.

2.1 Kinematic orbit estimation applying the raw observation approach

The raw observation approach has been introduced by Zehentner and Mayer-Gürr (2014), Zehentner and Mayer-Gürr (2016), Zehentner (2016). The basic principle of the raw observation approach is to introduce the GPS observations undifferenced and unchanged. This means every observation made by the on-board GPS receiver is used as it was observed, without forming differences or linear combinations, e.g. the ionosphere-free combination. Errors and influences acting on the measurements are taken into account by either estimation of a corresponding parameter or by subtracting known corrections. For example the ionospheric influence is considered by setting up a parameter for the slant total electron content (STEC). Parts which can not be estimated from the observations of a single GNSS receiver must be known in advance with highest possible accuracy. In conclusion it can

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be said that the raw observation approach for LEO orbit positioning can be considered as a precise point positioning approach (PPP).
The basic observation equation for the raw observation approach can be formulated as follows for code

$$R_{ri}^t - \Delta R_{ri}^t = \rho_r^t + c\delta t_r + \Delta I_{R_i} STEC + ACV_{ri}^R + ACV_i^{tR} \tag{1}$$

and phase observations

$$\Phi_{ri}^t - \Delta \Phi_{ri}^t = \rho_r^t + \lambda_i n_{ri}^t + c\delta t_r + \Delta I_{\Phi_i} STEC + ACV_{ri}^{\Phi} + ACV_i^{t\Phi}. \tag{2}$$

R and Φ are the original code and phase observations and the indices r, t, and i denote the receiver, the transmitter , and the carrier frequency. ΔR and $\Delta\Phi$ are the summed up a priori corrections for each observation. ρ is the range, containing the unknown receiver position and the known transmitter position. Furthermore, the equation contains δt_r the receiver clock error multiplied by the speed of light c, the $STEC$, and parameters for receiver and transmitter antenna center variations (ACV) for each observation type and frequency. For phase observations the relation is extended by the ambiguity term $\lambda_i n_{ri}^t$, where λ is the carrier wavelength and n the unknown number of cycles between receiver and transmitter.
The above mentioned observation equation is used to set up a least squares adjustment to estimate the sought for parameters. The implementation maintained at the IfG includes an iterative observation weighting scheme utilizing variance component estimation to detect outliers and reduce their weight. Usually orbit determination for LEO satellites is done on a daily basis. To reliably estimate antenna center variations for receivers as well as transmitters, it is necessary to process and combine a longer time span than a day. In fact months or even years shall be combined. Due to computational limitations it is not possible to directly process such long time spans. Due to this fact, antenna center variations must be estimated beforehand in a separate processing step. It is done by setting up the ACV parameters on a daily basis. Other parameters are eliminated by means of parameter elimination and the daily normals equations are stored. Finally all normal equations of a long time span are accumulated and solved. This procedure is repeated for receiver ACVs and transmitter ACVs separately. The estimated ACVs are subsequently introduced as known corrections and added to the left hand side of equation 1 and 2. The azimuth and nadir angle dependent parameterization for transmitter ACVs is shortly described in the following section.

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2.2 Azimuth and nadir dependent transmitter antenna center variations

Currently it is common practice to use the antenna correction provided by the IGS, which are only nadir angle dependent and identical for all satellites of one block. In contrast to state-of-the-art orbit processing approaches, we use individual azimuth and nadir angle dependent ACVs for all transmitters. As parameterization radial basis functions on a regular grid are used. To maintain an almost equidistant separation of the basis function, even close to the nadir direction, the triangle vertex distribution is used. A detailed description of radial basis functions as well as the triangle vertex distribution is given by Eicker (2008).
The parameterization of the ACVs as the sum of radial basis functions can be written as

$$ACV(\alpha, E) = \sum_{i=1}^{I} a_i \Phi_i(\cos\psi) \tag{3}$$

where

$$\Phi_i(\cos\psi) = \sum_{n=2}^{N} \sqrt{2n+1} \cdot P_n(\cos\psi). \tag{4}$$

The ACVs are modeled in dependence of azimuth α and elevation E (or nadir angle). More specifically, they depend on the spherical distance ψ between the actual point and the location of the base function. The base functions Φ_i are represented by a sum of Legendre polynomials P_n in combination with the specific coefficient a_i. i, the index of the base functions, runs from 1 up to the total number of

Figure 1: Gravity field results using different ACV configurations. blue: without ACVs, red: receiver ACVs and IGS transmitter ACVs, green: receiver ACVs and azimuth and nadir angle dependent transmitter ACVs.

considered base functions I. The summation in equation 4 only starts at degree 2, due to the fact that degree 0 and 1 are not distinguishable from a scale or shift in the satellite position.

The effect of azimuth and nadir angle dependent ACVs on derived gravity field solutions has been shown by Zehentner and Mayer-Gürr (2014), Zehentner (2016). Figure 1 shows three different versions of monthly gravity field solutions derived from kinematic orbits of GRACE A for January 2012. Version 1 utilizes no ACVs for receiver and transmitters. Version 2 uses estimated receiver ACVs in combination with transmitter ACVs provided by the IGS. In version 3 the IGS ACVs are replaced by azimuth and nadir angle dependent ACVs, provided individually for each transmitter. Version 1 is not of interest at this stage. However, the improvement achieved from version 2 to 3 is clearly visible in low degrees (2-15). In addition a slight improvement can be seen over the whole spectrum.

This comparison demonstrates the benefit which could be gained from using a consistent set of ACVs for receiver and transmitter. In addition it shows the potential of azimuth and nadir angle dependent individual ACVs for each transmitter. In contrast to state-of-the-art processing approaches we also include ACVs for code observations, which improves their accuracy and enlarges their contribution to the orbit solution.

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2.3 Azimuth and elevation dependent observation weighting

In GNSS processing it is common practice to utilize a fixed observation weighting or some sort of elevation dependent approach. We propose to use an azimuth and nadir angle dependent observation weighting scheme in conjunction with variance component estimation to detect outliers and reduce their weight. In order to realistically describe the error behavior of a certain receiver-antenna configuration we analyze observation residuals of a longer time span.

The antenna hemisphere is divided into regular grid cells and each residual is assigned to a cell according to its azimuth and elevation. All residuals assigned to a cell are then used to compute an empirical standard deviation for this cell. The corresponding redundancy of each residual is taken into account. This is done for the whole hemisphere and for each observation type individually. This way a map is produced describing the standard deviation of an observation depending on azimuth and elevation. The approach is described in detail in Zehentner (2016). Figure 2 shows accuracy maps for GRACE A and GOCE for L1 phase observation.

Figure 2: Accuracy maps for L1 phase observation of GOCE (left) and Grace A (right).

Figure 3: Gravity field results using different ACV configurations and weighting schemes. blue: without ACVs, red: receiver ACVs and IGS transmitter ACVs, green: receiver ACVs and azimuth and nadir angle dependent transmitter ACVs, purple: same as version 3 with azimuth and elevation dependent weighting scheme.

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As it can be seen the accuracy of L1 phase observations does vary over the whole hemisphere and especially for GOCE it is not only elevation dependent. In contrast to GRACE A, on the right panel, GOCE shows rather strong variations distributed arbitrarily. The effect on the estimated gravity field is shown by Zehentner (2016) and in Figure 3. Figure 3 shows the same three monthly solutions as already shown in Figure 1 and an additional fourth solution is added. It uses the same configuration of ACVs as solution 3 but uses the azimuth and elevation dependent observation weighting scheme. For version 1 to 3 fixed a priori weights have been used. It can be seen that version 4 outperforms version 3 over the whole spectrum. This comparison is based on kinematic orbits from GRACE A, for which the variation in the empirical standard deviation is rather small and mainly elevation dependent. From Figure 2 it can be seen that for GOCE the variation is much more inhomogeneous and thus a comparable or even bigger improvement can be expected.

2.4 Observation weighting using SNR or TEC information

In addition to a realistic description of the theoretical observation quality it would be beneficial to have information about the actual quality of each observation. Most GPS receivers do provide such information in terms of the signal to noise ration (SNR) contained in the observation files (RINEX) as additional observation type or as a normed quality indicator. This information could be used to determine an individual weight for each observation. Investigations conducted at the IfG and published in Zehentner (2016) have shown that this information could contribute to a better gravity

Figure 4: Difference in geoid height between monthly GOCE solutions for October 2010 and GOCO05S. Left: no SNR information was used, right: SNR values used to scale the observation weight. 500 km Gaussian filter applied..

field solution.

The SNR information was used to scale the a priori weights of each observation by using a simple scale function. The derived kinematic orbit was then used to estimate a monthly gravity field and compared to a solution without applying the scale function. Figure 4 shows geoid height differences with respect to the static field GOCO05S with a 500km Gaussian filter applied. It can be clearly seen that the effect along the magnetic equator is greatly reduced, although it is not fully removed. The overall RMS of difference to the static field reduces from 6.6 mm to 5.6 mm. In this case the RMS also includes time variable signals of the gravity field, as only a static field has been subtracted.

The drawback of SNR values is the fact that they only reflect the signal quality at the receiving antenna. Influences which do not affect the signal strength and quality are not reflected in the SNR values. Errors induced by ionospheric irregularities, causing the gravity field error along the geomagnetic equator, can be described by various ionospheric indices. One of them is the rate of TEC index (ROTI) (Pi et al., 1997) defined as the variance of the rate of TEC in a predefined time window. It is a way to describe short term variations in the ionosphere. The main advantage of the ROTI is the fact that it can be derived from the observations themselves. Again a simple scale function has been defined to scale the a priori weights. Figure 5 shows the same comparison as Figure 4. Again a clear reduction of the effect along the geomagnetic equator can be seen. The error is not fully removed but again the overall RMS is reduced to 5.6 mm.

Both approaches show the benefit of a realistic description of the observation quality and demonstrate that the error along the geomagnetic equator can be greatly reduced. Further investigations should be conducted to find the best way to utilize this type of information in the process of kinematic orbit estimation.

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Figure 5: Difference in geoid height between monthly GOCE solutions for October 2010 and GOCO05S. Left: no ROTI scale function applied, right: ROTI scale function used to scale the observation weight. 500 km Gaussian filter applied..

3 Studies and strategies to improve the gravity gradient processing

As a result of the discussion in the WP6000 team and concluded from studies performed with contributions by the team members, two major lines of thoughts were identified, where remaining potential can be extracted from the GOCE gravity gradient data.

The first line of thoughts directly follows the 'classical' time-wise processing method and tries to improve the individual processing steps in the treatment of the L1B gravity gradient observations. Staring with the L1B product for the gravity gradients, the functional model is straight forward and does not leave room for improvements. The main aspects in the gradiometer data processing, where known simplifications are made and room for improvements is seen covers the uncertainty description/modelling of the measurements. This includes on the one hand the treatment and identification of *outliers*, *systematic effects* (e.g. oscillations in the data) or any kind of other *suspicious data*. On the other hand, this includes the modeling of the stochastic characteristics of the gravity gradients. Due to the measurement principle, they are highly correlated. Because of the huge number of observations, a simplifying model to approximate its covariance has to be introduced. Due to the mission design, an imperfect calibration and the changing environment of the spacecraft, the stochastic characteristics seem to change during the mission. Thus reconsidering or improving the stochastic modeling, especially the change of the stochastic model in time, leads to an improvement of the gravity field estimates.

The second line of thought for possible improvements picks up a quite old study from Mayer-Gürr (2011), which was not continued anymore since then. Nevertheless, the results presented there were very promising, using an alternative approach to process the gradiometer observations and it seems worth looking into the idea again. Basically, instead of using the calibrated L1B gradiometer data (EGG_NOM product), the accelerometer observations are directly used for gravity field determination. The calibration (as performed independently for L1B gradiometer data) is directly co-estimated in the least-squares adjustment of the gravity field. Results of the alternative processing can be used to i) verify the current gravity field solutions, ii) verify the calibration of the gravity gradient and detect if there is remaining potential and iii) possibly yield better global gravity field estimates. The big advantage of the approach is that the calibration parameters can be estimated/updated with an arbitrary temporal resolution to account for temporal changes. Furthermore, an arbitrary deterministic model can be used to describe the temporal changes during the validity perod of a set of calibration parameters (e.g. linear drift, quadratic term, ...).

3.1 Strategies inline with the 'classical' time-wise processing

Within the studies during and after the generation of the EGM_TIM_RL05 model (Brockmann, 2014, Brockmann et al., 2014) the following possible improvements for the gradiometer processing

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Figure 6: Time series of the GOCE observations along the entire mission. Colors represents parts of the time series without a data gap.

were identified, which lead to an updated and improved global gravity field model, if implemented within a reprocessing campaign.

3.1.1 Estimation of decorrelation filters for the gravity gradients as stochastic model

3.1.1.1 Used approach for EGM_TIM_RL01 to EGM_TIM_RL05

Within the official releases RL01–RL05, cascaded Auto-Regressive Moving-Average (ARMA) filters were estimated from the data (residuals, observed minus computed) and used in the least-squares gravity field adjustment as decorrelation filters (or whitening filters, or as stochastic model) for the L1B gravity gradients which are introduced as observations. The basic idea is to estimate the ARMA filter coefficients from the data to achieve a data adaptive decorrelation, which accounts for changes in the characteristics of the observations(see e.g. Siemes, 2008, Schuh et al., 2010, Krasbutter et al., 2011) within the time series. For that reason, within the processing of the five models, the entire available observation time series is partioned into gap less segments, cf. Fig. 6. For the entire missions $S = 87$ segments were used.

From the observations from each of the segments $s \in \{1, ..., S\}$ and for the individual gravity gradient components $c \in \{XX, XZYY, ZZ\}$ an individual decorrelation filter $f_{s,c}$ was estimated from observation residuals $\mathbf{v}_{s_i,c_j}$ after a least-squares gravity field adjustment. For the final RL05 model which covers the entire time series, $4 \cdot 87 = 348$ filters were estimated and used. Within a segment s_i and for a specific component c_j, the filter f_{s_i,c_j} is assumed to be time invariant. The filter coefficients were determined from the residuals in a least-squares adjustment of an ARMA process. For that purpose, a preferably long but quiet time sector – which is not obviously influenced by outliers, artifacts, satellite maneuvers, and so on – was manually selected for each of the segments, to estimate the filter for the segment (see for instance Fig. 7).

To estimate the filter with the used approach (see Siemes, 2008), all residuals of that part are used with equal weights to derive a least-squares solution of the filter, which then serves as a "mean" filter for the entire segment s_i. Fig. 7 shows an example segment, where the residuals used for filter estimation are shown in red. Using such an ARMA model implicitly assumes (due to the filter model properties) that the approximated time series is stationary. That means, the covariance/correlation does not depend on the location of the observations but only on the time lag in the time series. As can be seen directly from Fig. 7, the data set contains some measurements which seem to have different characteristics. We can call this data points *outliers* or *gross errors*, but also some other data characteristics which we might call *suspicious data*, which do not seem to be outliers, but data with other noise characteristics than the rest of the segment. These parts of the time series destroy the stationarity of the time series.

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Figure 7: Observation residuals for segment $s = 55$ and all components XX and YY (on week in November 2012).The red section is used for filter estimation, the blue parts were neglected for the filter estimation.

Figure 8: Zoom to decorrelated GOCE observation residuals for a segment and component.Non stationary characteristics of the time series are obvious.

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These effects can be nicely shown after a decorrelation of the residual time series, for instance as shown in Fig. 8 or later in Fig. 10. Non-stationary effects in the decorrelated time-series are obvious. Such small effects are not visible in the "raw" residual time series, and are thus included in the time series the filters are estimated from. Many of these effects are once per revolution, so that each long time series is influenced – more or less – by non-stationarity. Due the characteristics of the data, these non-stationarity leaks into all frequencies of the filter and thus result in a degraded (to pessimistic!!!) filter for the entire frequency band. Thus, the spectral weighting is most of the time to pessimistic. A perfect combination of different segments and the different gravity gradient components can not be possible with that filters. So whenever segments and/or components are combined, the solution will not be optimal.

3.1.1.2 Use of robustly estimated decorrelation filters to deal with non-stationarity of the time series

To improve the estimated filters and to minimize the effect discussed before, a study was performed on the estimation of the decorrelation filters such that they are estimated independently of the gross-errors, suspicious data and the manual selection of the estimation time period Schuh et al. (2015). The idea is very simple and straightforward, instead of an least-squares estimation from a manually selected "good" sub-segment, a robust estimation procedure is performed based an the entire data

Figure 9: Observation residuals for segment $s = 55$ and components XX and YY (on week in November 2012).The red shows identified outliers, the blue data is used to estimate the filters.

available for the segment.

Methodology Thus, using for instance a robust L_1-norm estimator, robustly estimated filters can be derived $f^r_{s,c}$ for each segment and each component. Using the robust estimation, the manual selection of the "long put quiet time sector which is not obviously influenced by outliers..." can be avoided. Instead, the robust estimator automatically derives the filter and in addition classifies the "good" data which is consistent to the robustly estimated filters. The robust approach was studied and presented in Schuh et al. (2015). Whereas in the previous processing ARMA filers were used (around order 50 for AR and MA part), this method is very well suited for AR filters. As the recursive part is missing, the order of the used AR processes is higher (around 800). Nevertheless, the filters are of comparable complexity (ARMA filters can be always converted to longer AR filters). Within the robust estimation step, an AR process is adjusted to all of the residuals of a segment (and a component). As a supplementary product, outlier information is returned by the estimator (weights of the iteratively re-weighted least-squares principle often used to find the robust solution). As an example, the 'outliers' or the 'suspicious data' identified by the robust estimation procedure are shown as red dots for segment 55 in Fig. 9. Compared to Fig. 7, it is obvious that many of the 'suspicious data' entered the old filter estimation.
Looking into the decorrelated residuals for that period (cf. Fig. 10), it becomes obvious, that the identified data are not only outliers. It seems like there are changes in the noise characteristics, periods in time where the noise increases. In addition some oscillations are visible.

segment	start	end	# observations	note
50	01-Aug-2012	31-Aug-2012	2 645 205	first decay phase
58	02-Dec-2012	13-Jan-2013	3 632 149	
76	17-Jun-2013	13-Jul-2013	2 325 772	

Table 1: Test segments used to study and compare the new filter estimation procedure (Brockmann et al., 2015a).

New robustly estimated filters As the theory seems to work, a systematic study was performed to identify if the decorrelation filters are useful for gravity field determination from the gravity gradient data. In Brockmann et al. (2015a) the robustly estimated filters were used to compare the 'classical' filter estimates with the newly derived filter estimates. Based on three selected test segments (cf. Tab. 1) with different characteristics, it was studied how the newly derived filters perform for gravity

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Figure 10: Decorrelated/filtered observation residuals for segment $s = 55$ and all components (on week in November 2012).The red shows identified outliers, the blue data is used to estimate the filters.

field recovery and how the outlier/suspicious data detection works. All chosen test segments are from the extended mission phase, where GOCE was measuring in a lower orbit and thus in a rougher environment. The filters and later on the solutions from the official EGM_TIM_RL05 processing serve as a comparison.

Within the new approach, all data which does not fit to the estimated filter model are treated as outliers and are excluded from the gravity field recovery. In the old approach, all decorrelated residuals above a threshold and a certain range around outliers were flagged and not used. Compared to the approach used in the RL05 model generation, more outliers are identified (cf. Tab. 2). Fig. 11 shows the geographical distribution for the outliers identified for segment 76 by the robust filter estimation approach. Especially for the YY-component, the correlation to the magnetic poles is visible. The new method is more aggressive w.r.t. outliers and only selects the highest quality data for filter estimation. To be consistent, only this data is used later on for gravity field recovery.

The estimated filters determined with both approaches for the three test segments are shown in terms of their PSD in Fig. 12. The results and conclusions of Fig. 12 are not surprising. As only the 'best' data is selected for the estimation of the decorrelation filters, the noise level of the new filters is generally lower, at least it is on on the same level as for the old filters (for all components, segments and most of the frequencies). In addition, the new filters mostly show a flatter characteristic in the measurement band. This is especially observable for the lower end of the measurement band of the YY component, followed by XZ, XX and ZZ component. Especially for segment 50, large improvements are visible, but note that this is a special segment, i.e. the first decay phase of the satellite. To conclude the analysis of the estimated filters one can say that the new robustly estimated filters suggest that data is more accurate than old filters assume. As especially the low orbit data loose stationarity (higher noise at special locations due to rougher environment) the 'old' filters are degraded by the non-stationarity. Now, the new approach concentrates on stationary data for filter estimation, but has

segment	XX		XZ		YY		ZZ	
	#	%	#	%	#	%	#	%
50 old	70 600	2.7	35 036	1.3	5 531	0.2	42 879	1.6
50 new	707 381	26.7	613 017	23.2	298 166	11.3	33 839	1.3
58 old	51 335	1.4	100 429	2.8	10 129	0.3	40 089	1.1
58 new	28 066	0.8	15 704	0.4	398 071	11.0	6 924	0.2
76 old	78 006	3.4	102 083	4.4	221 230	9.5	13 149	0.6
76 new	160 428	6.9	61 415	2.6	584 366	25.1	141 187	6.1

Table 2: Statistics on identified outliers for the three segments used in Brockmann et al. (2015a) for the new and old approach.

(a) $s = 76$, $c = XX$ (b) $s = 76$, $c = XX$

(c) $s = 76$, $c = XX$ (d) $s = 76$, $c = XX$

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Figure 11: Distribution of the outliers identified by the robust filter estimation approach for the test segment $s = 76$ and for all 4 components.

(a) $s = 50$, $c = XX$ (b) $s = 58$, $c = XX$ (c) $s = 76$, $c = XX$

(d) $s = 50$, $c = XZ$ (e) $s = 58$, $c = XZ$ (f) $s = 76$, $c = XZ$

(g) $s = 50$, $c = YY$ (h) $s = 58$, $c = YY$ (i) $s = 76$, $c = YY$

(j) $s = 50$, $c = ZZ$ (k) $s = 58$, $c = ZZ$ (l) $s = 76$, $c = ZZ$

Figure 12: PSD of filters estimated with the new approach (green) compared to the old filters used in EGM_TIM_RL05 for the three test segments and all four components.

the drawback that data which does not agree with filter model are marked as outliers. Based on this findings, it can not be concluded if the new filters are better suited for gravity field determination. So it has to be studied, how the new filters perform in the gravity field determination procedures.

Gravity field solutions for the three test segments For this three test segments, a systematic case study was performed in Brockmann et al. (2015a). Gravity field solutions determined based on the gravity gradient observations were computed for each of the three segments. For every segment, individual solutions per component and a combined solution from all four components (XX, XZ, YY and ZZ) were computed (if the normal equations were solvable i.e. positive definite). The same solutions were available from the official RL05 processing. Both solution families (new vs. old) differ by i) the outlier handling (RL05 vs. information from robust filter estimation) and ii) in the used filters (RL05 via new robust estimation). Though the solutions are based on a different number of observations (cf. Tab. 2). In most of the cases, less observations were used for the new solution.

Fig. 13 collects the three times five estimated gradiometer-only solutions in terms of degree variances. As compiled from much more data, EGM_TIM_RL05 serves as a reference model. Shown are always differences to the RL05 model (solid lines) and the corresponding formal errors (dashed lines). The solutions computed with the old filters are shown in red and the solutions with the new filter in green. As combined and component-wise solutions are studied, the study is very well suited to see the influence of the new filters on the different gravity gradient components, the general performance of the newly derived filters and on the combination of the different components.

The solutions for segment 50 are shown in the first column of Fig. 13. This segment is influenced by the first decay phase of the satellite. The solutions for XX and XZ seem to be strongly influenced by the drag bias. For XX the old solution is very bad, hardly visible in the range limits of the plot. The new method identified 25 % of outliers, resulting in to few observations to solve the new XX normal equations for that segment. For XZ the same amount of outliers were identified, 20 times more then for the old solution. Consequently the individual new solution is of a higher quality. For this segment and component the new solution is worse. For segment 76, component YY the new method again identified around 25 % of outliers, resulting in to few observations to solve the new YY normal equations for that segment.

The other conclusions and results can be drawn together for every component. The formal error estimates for the computable YY-only solutions significantly reduced and agree much better to the empirical differences (degree 10 to 180). This is best visible for the YY component, which is largest influenced by the non-stationarity. In addition, the solution itself slightly improved (degree 10 to 80). The same holds true for the computable XX, remaining XZ and ZZ solution, but with a much smaller extend. To sum it up quite general: small to tiny improvements for the solutions big to small improvements for the formal error estimates when using the robustly estimated filters for individual components.

Nearly the same conclusions can be drawn for the combined solutions. For all three segments the formal errors are lower and better correspond to the difference. The solutions itself are all better and do better agree to RL05. For the combined solution, the improvement can be quantified with cumulative degree error variances for each segment (last row in Fig. 13):

- $s = 50$: cumulative formal error for degrees 30–200 reduces by 24 % (deg. 30–280→reduction 16%) cumulative empirical error for degrees 30–200 reduces by 20 % (deg. 30–280→reduction 9%)
- $s = 58$: cumulative formal error for degrees 30–200 reduces by 12 % (deg. 30–280→reduction 9%) cumulative empirical error for degrees 30–200 reduces by 10 % (deg. 30–280→reduction 6%)
- $s = 76$: cumulative formal error for degrees 30–200 reduces by 13 % (deg. 30–280→reduction 8%) cumulative empirical error for degrees 30–200 reduces by 7 % (deg. 30–280→reduction 5%)

Although for all three segments less observations were used, the combined solution for all segments improves in the order of 10–25 % for the formal errors and 7–20 % for the comparison to RL05 model. Depending on the amount of suspicious data in the segment, the gain is larger. The main conclusion is that the new approach improves the gravity field estimates and delivers better and more realistic

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(a) $s = 50$, $c = XX$

(b) $s = 58$, $c = XX$

(c) $s = 76$, $c = XX$

(d) $s = 50$, $c = XZ$

(e) $s = 58$, $c = XZ$

(f) $s = 76$, $c = XZ$

(g) $s = 50$, $c = YY$

(h) $s = 58$, $c = YY$

(i) $s = 76$, $c = YY$

(j) $s = 50$, $c = ZZ$

(k) $s = 58$, $c = ZZ$

(l) $s = 76$, $c = ZZ$

(m) $s = 50$, $c = XX$, XZ, YY, ZZ

(n) $s = 58$, $c = XX$, XZ, YY, ZZ

(o) $s = 76$, $c = XX$, XZ, YY, ZZ

Figure 13: Segment-wise gravity field solutions for the three test segments in terms of degree (error) variances. Red shows the solutions where the old RL05 filters are used, green colors show the solutions where the new robust filters where used. Shown are solutions per component and combined solutions compared to the superior RL05 model. Note that for segment 50 XX no individual solution was computable. As for XZ about 25 % outliers were identified the quality of the solution is bad, but as bad as reflected by the formal errors. No solution was computable for segment 76, c=YY due to the short time series and the large number of outliers.

covariance estimates. Although the individual solutions itself hardly improve, their covariance information improves. Thus, due to a better frequency depending relative weighting of the components, the combined solutions improve. Espically the important YY component is weighted higher in the spectral bands as indicated by Fig. 13. To sum up the results of the study (cf. Brockmann et al., 2015a), it follows that the gravity field solutions (for individual segments & components) with robustly estimated filters show remaining potential in GOCE data:

- The case study has shown: positive effect for lower orbit data
- There is strong impact when combining components and segments (improved relative weighting due to more realistic formal errors)
- The strengths of individual components are better kept in combination
- Although less observations are used, no negative effect visible in new solutions, improvements for entire spectrum
- Improvements for individual segments is in the range of 5–20%

Preliminary gravity field solutions from the entire mission data set The test segments used above were all from the extended mission phase after/during the orbit lowering campaigns. Thus, as the non-stationarity of the observations is expected to be larger there (compared to the nominal mission data), from the above mentioned study the gain for an entire reprocessing could not be concluded. But, motivated from the result, a kind of 'brute force' processing for the entire time series was performed in Brockmann et al. (2015b). Basically, the software to estimate the robust filters from the residuals used in the study above was applied to estimate robust filters for all segments of the time series and all for all four components. Without verifying the estimated filters and outlier information in detail, the output was used to produce the a preliminary global gravity field model from the entire mission dataset. Note that for some segments and components no robust filter was estimable (in total about 10 filters failed), the resulting filters were numerically instable and not usable for gravity field determination. For that components and segments the RL05 filters were used in the study. The reason why the filters were not estimable were not finally studied and not yet verified.

Normal equations for the entire mission dataset were computed, combined and solved to obtain a reprocessed SGG-only gravity field solution using the newly estimated robust filters and the updated outlier information from the filter estimation process. Within the entire time series in RL05 2.6 % outliers were identified, the updated approach identifies two times more, i.e. 5.3 % outliers. The updated gradiometer-only solution is shown in terms of degree (error) variances compared to EGM2008 in Fig. 14. Shown are three solutions, a) a solution computed only from the nominal mission data (RL04), b) a solution only from the extended mission data (all data after RL04) and c) the solution from the entire mission time series. The plot indicates that all three solutions with the new filters are better and improved. The improvements for the lower degree for the nominal mission data seem to be larger compared to the extended mission data. In contrast to that the high degree improvements are larger for the extended mission data compared to the improvements from the nominal mission data. The improvements for the formal errors can be quantified for degrees 20 to 200, for the nominal mission data it is 9.6 % reduction, for the extended mission data it is 16.1 % reduction, and for the entire mission it is 12.5 % reduction.

Fig. 14(c) shows the improvement analyzing the entire data set. Although 2.6 percent less data are used within the new solution, an improvement is seen for the entire spectrum (where EGM2008 has a good performance, degrees 2–50 and 220/230+). In this areas, formal and empirical errors agree quite well. From the gradiometer only solution, the improvements can be only quantified using the formal error estimates, for degrees 20–200 a reduction of 12.5 % and for degrees 20–280 a reduction of 9.9 % is seen. To be more flexible, the new solution was estimated to degree and order 300, but no study was performed to demonstrate if that higher degrees contain local signal.

To conclude the study, one can say that:

- The case study has shown: positive effect for the entire mission data

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(a) Data from nominal mission only (RL04)

(b) Data from extended mission only (RL04-RL05)

(c) Data from entire mission only (RL05)

Figure 14: Gradiometer-only solutions with the old and new filter for first part of the mission, extended mission and entire mission compared to EGM2008.

- There is stronger impact for lower orbit data
- The strengths of individual components and segments are better kept in combination
- Although less observations are used, no negative effect visible in new solutions, improvements for entire spectrum
- Improvements in the range of 10.0 %
- There is remaining potential in GOCE data!
- See also later Sect. 5 where a combined solution is studied additional validations are provided there

Some aspects have not been studied yet and might yield to even more improvements but require a more sophisticated reprocessing:

- The outlier information is not verified. The output of the filter estimation is used without an analysis.

- Instead of flagging observations identified as outliers or better suspicious data, it might be down-weighted instead of not used (Strong geographical correlations to e.g. magnetic field, flagging of additional gaps into the time series).
- Some filter estimates fail, the reason is not understood, bad convergence, numerical properties and so on. A lot of work is required to understand and optimize the filter estimation process.

3.1.1.3 Time-variable filter estimates The studies above have shown, that there are 'outliers' and 'suspicious data' in the GOCE gravity gradient time-series which have an influence on the stochastic model and the decorrelation filters. The robust estimation filter approach studied above has the big disadvantage, that all data which do not agree with the filter model are treated as outliers, no matter if they are real outliers or parts of the time series with different characteristics. To improve the situation, the study presented in Schuh and Brockmann (2015) tries to extend the AR filter model to a non-stationary model. So instead of identifying observations which do not agree to the model, the decorrelation filter model is made time-variable to allow a change of the characteristics within a segment s.

The idea is to make every AR coefficient $\alpha_j^s(t)$ time dependent, such that for any time t another filter is used. So every filter coefficient is a function of time. The observation equation to estimate the filter coefficients reads

$$l_i + v_i = \sum_{j=1}^{p} \alpha_j(t_i) l_{i-j} = [l_{i-1} \dots l_{k-p}] \begin{bmatrix} \alpha_1(t_i) \\ \vdots \\ \alpha_p(t_i) \end{bmatrix} \tag{5}$$

$$:= \boldsymbol{a}_i^T \boldsymbol{\alpha}(t). \tag{6}$$

To allow for stable estimates of the temporal characteristics, the function $\alpha_j(t)$ is set up as a deterministic model. General B-splines are setup at predefined nodal points in time, such that the actual filter $\alpha_j(t)$ at time t is composed by the sum of local deterministic base functions at an arbitrary time by

$$\alpha_j(t) = \sum_{k=1}^{K} a_{jk} B_k^g(t). \tag{7}$$

So the unknown parameters estimated in the adjustment are the weights a_{jk} for the Spline base functions. These can be estimated from the gravity gradient residuals l_i for every segment and component. Based on the weighting of the B-spline functions, the AR filters follow at an arbitrary point in time. So far, the weights a_{jk} were estimated in an least-squares adjustment.

To apply the approach to the filter estimation from GOCE gravity gradient residuals, the definition of the nodes for the B-spline base function is important. In a first study, Schuh and Brockmann (2015) placed the nodes once a revolution, where the 'suspicious' data is expected. Around that area three nodes are placed to allow for a transition zone (See Fig. 15(i)). This approach was implemented and tested in a small test scenario (16 June 2012 2:59:53 — 16 June 2012 21:46.56, twelve orbit revolutions). So far, only the YY-component is studied, as the data are mainly affected by the changing characteristics. In the first row of Fig. 15 the residuals of the measurements with respect to the EGM_TIM_RL05 model are plotted. The suspicious data for the last orbital revolutions are obviously visible. For that data period the time variable filters are estimated and the decorrelation process is applied to the residuals. A comparison to the 'classical' approach (cf. 3.1.1.1) and the robust approach (cf. 3.1.1.2) is shown in Fig. 15 in different representations. The first column shows the plots for the classical approach, the second for the robust approach and the third for the time-variable approach.

Whereas the first two rows only introduce the input residual data (extended by identified outliers for the robust method) – which is the same for the three approaches – the 3rd to 5th row show the results to be compared. The results of this new method are studied only for the decorrelation so far, the time-variable filters were not yet used for gravity field recovery. The third row shows the success of

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(a) RL05 residuals for YY

(b) RL05 residuals for YY

(c) RL05 residuals for YY

(d) RL05 residuals for YY – zoom

(e) RL05 residuals for YY – zoom

(f) RL05 residuals for YY – zoom

(g) filtered res YY – standard

(h) filtered res YY – robust

(i) filtered res YY – time variable

(j) PSD filtered res – standard

(k) PSD filtered res – robust

(l) PSD filtered res – time variable

(m) BSD res & filter – standard

(n) BSD res & filter – robust

(o) PSD res & filter – time variable

Figure 15: Results of the 3 different filter estimation procedures and its effects on the decorrelated residuals. The 1st column shows the 'classical' approach used in RL05, the 2nd the robust filter and the 3rd the time variable filter. Shown are residuals and decorrelated residuals in the time and frequency domain. The comparison is shown for a part of a test segment (YY-only).

the decorrelation, the filtered residuals, which should be ideally random white noise with zero mean. For the classic approach, the suspicious data remain clearly visible, the amplitude is damped a little. The robust approach keeps the systematic structure, but identifies the data period where the effects occur as outliers. Using the time variable filters removes the effects during filtering, no systematic effects remain visible. This is supported by the fifth row, where the PSD of the filtered residuals are shown. The first approach shows remaining frequencies after decorrelation at around $2 \cdot 10^{-3}$ Hz, the robust approach leads to a close to white noise spectrum excluding the suspicious data. The spectrum of the time variable filtered noise is closest to white noise at demonstrates that the decorrelation is properly working. The final row shows the spectrum of the correlated noise and the spectrum of the filter which approximates the noise spectrum. The degradation of the spectrum due to the suspicious data is clearly visible. For the time-variable filters a range of spectra is shown, showing the time variable filters for different points in time. The indicates the variation of the filters captured by the time variable model.

The study uses just residuals and decorrelated residuals to verify the approach. The filter was estimated for a simplified set of nodes for the time-splines. Nevertheless, the decorrelation clearly shows that the 'suspicious' data characteristics in the data can be captured by time variable AR filters. The data can be used in the adjustment and the filter correctly descibes the characteristics. So an improvement for gravity field determination is expected, even compared to the improvements achieved with the robustly estimated filters. Due to the variability of the filter model, the entire stochastic characteristics are expected to be better captured for the entire time series. This study was not performed yet, as there are some open points which must be solved before an automated time variable filter estimation can be performed, which is the prerequisite for studying the influence on gravity field solutions. The open questions and points which have to be addressed and solved cover:

- The placement of the nodes of the B-Splines, which determine where strong changes of the filter are allowed. They have to be automatically identified in an data-adaptive way. For instance the outlier information derived with the robust approach may serve as a good proxy where to blace the B-splines.
- The approach was so far only studied for the shown segment, as the filter estimation and analysis is very time consuming. In addition it was only tested for the strongest influenced component YY. Portability to other data segments and the other segments have to be demonstrated.
- The algorithms for the decorrelation step within the gravity field determination have to significantly updated before these filters can be used for gravity field determination. As currently time-invariant filters are used, very efficient decorrelation can be done as described in Brockmann (2014), using the fact that the filter coefficients have be constant so far (for an entire segment).
- The numerical stability of the entire set of 'temporal' filters must be verified before the gravity field adjustment is entered.
- The 'success of the decorrelation' should be statistically tested to decide if the nodes are correctly placed.

Despite this open points, a better and more consistent SGG-only solution is expected from this advanced modelling of decorrelation filters, capturing so far non-visible changes in the observation characteristics.

3.1.2 Outlier analysis and data usage

Depending on the finally chosen decorrelation filter estimation the outlier identification has to be chosen. In addition to the suspicious data, real outliers resulting from instrument failure, maneuvers and so on remain even after a time-variable decorrelation in the data set. Within the time-wise processing, the idea is always to start with the entire data set available, iteratively identifying observations as outliers or gross errors which do not agree to the setup. At that stage, no a-priori information from outside the GOCE mission should be used, as the time-wise models should represent the gravity field as observed by GOCE. So outliers or gross errors have to be identified from the filtered residuals

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iteratively. As the filtering smears an outlier throughout the time-series, the identification of a single outlier is tough.

3.2 Novel strategies outside the 'classical' time-wise approach

Some years ago, Mayer-Gürr (2011) presented an alternative approach to estimate a gravity field from GOCE observations. Instead of using the calibrated L1B gravity gradients (EGG_NOM product), the accelerations measured by the six 3D accelerometers were directly used as input.

3.2.1 Brief overview and idea of the alternative processing method

Basically, the observation equation for the approach follows from the equation of motion in a rotating reference system as

$$\begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} = \underbrace{\begin{pmatrix} s_x & \alpha+\eta & -\beta+\delta \\ \alpha+\eta & s_y & \gamma+\chi \\ -\beta+\delta & \gamma+\chi & s_z \end{pmatrix}}_{\substack{\text{inverse calibration matrix} \\ \text{(scale, rotation, shear)}}} \left[\left[-\underbrace{\begin{pmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{pmatrix}}_{\text{gravity gradients}} + \underbrace{\begin{pmatrix} -\varpi_y^2-\varpi_z^2 & \varpi_x\varpi_y & \varpi_x\varpi_z \\ \varpi_x\varpi_y & -\varpi_x^2-\varpi_z^2 & \varpi_y\varpi_z \\ \varpi_x\varpi_z & \varpi_y\varpi_z & -\varpi_x^2-\varpi_y^2 \end{pmatrix}}_{\substack{\text{centrifugal force} \\ \text{(angular rates)}}} + \underbrace{\begin{pmatrix} 0 & -\dot{\varpi}_z & \dot{\varpi}_y \\ \dot{\varpi}_z & 0 & -\dot{\varpi}_x \\ -\dot{\varpi}_y & -\dot{\varpi}_x & 0 \end{pmatrix}}_{\substack{\text{Euler force} \\ \text{(angular acc.)}}} \right] \underbrace{\begin{pmatrix} L_x \\ L_y \\ L_z \end{pmatrix}}_{\text{position}} + \underbrace{\begin{pmatrix} c_x \\ c_y \\ c_z \end{pmatrix}}_{\substack{\text{non conservative forces} \\ \text{(common mode)}}} \right] \tag{8}$$

for the measurements of one out of the six accelerometers. Inserting the gravitational tensor **T** written in spherical harmonics and applying the rotation to the gradiometer reference frame, yields the observation equations, in which the following unknown parameters are set up

- Gravity gradients in form of the spherical harmonic coefficients (global)
- $6 \cdot 3$ accelerometer positions (global)
- $6 \cdot 9 \cdot 2$ calibration parameters and its linear drift (global for a predefined area of validity, can be newly estimated whenever it seems useful)
- 3 common mode accelerations $\begin{bmatrix} c_x & c_y & c_z \end{bmatrix}$ (per epoch)
- 3 angular accelerations $\begin{bmatrix} \dot{\omega}_x & \dot{\omega}_y & \dot{\omega}_z \end{bmatrix}$ (per epoch, plus start values per arc).

All of them can be estimated in a joint least-squares adjustment. Per epoch, 18 observations from the six accelerometers are used to estimate the parameters. In addition, the star camera observation can be used, which are sensitive in special spectral bands and help to estimate the angular rates and accelerations.
As can be seen from (8), the observation equations of this alternative approach are non-linear. For that reason, a linearization according to Taylor is required. As good and accurate approximate values exist (e.g. the L1B gravity gradients, GOCE-only gravity field models, official ICM, ...), the linearization is straight forward and iterations are not required. After setting up the observation equations, the rest of the processing is basically the same compared to the steps performed in the 'classical' time-wise approach:

1. Set up observation equations for short arcs ($\approx$ 15 min)
2. Set up a stochastic model for the acceleration observations (e.g. via filters or via their PSD)
3. Decorrelate the observation equations
4. Eliminate common mode and angular acceleration parameters from the system of equations
5. Accumulate normal equations of spherical harmonic and other parameters of interest (for instance calibration parameters) for the gradiometer observations
6. Combine with for instance SST normal equations, stabilize the solution and solve for the parameters

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Figure 16: Error PSDs of the 18 acceleration measurements.

3.2.2 A first test of the method

The proposed alternative processing strategy was tested in Mayer-Gürr (2011) for the first 71 days of observations (slightly more then the first cycle, the same as the official RL01 time period). Within that study, the accelerations where down-sampled to a 5 s sampling. The stochastic model is set up as using analytical cosine error covariance functions which weights were estimated from the data. That approach assumes (as the classical time-wise approach) stationarity, the independence of accelerometer axes and the independence of the different arcs. The PSD reflecting the error characteristics of the accelerometers are shown in Fig. 16. The different characteristics of the different accelerometers and accelerometer axes are visible.

The gravity field solution (gradiometry-only) computed in Mayer-Gürr (2011) at that stage, outperformed the official RL01 model (cf. Fig, 17). As seen in the plot, the alternative solution (marked as ITSG-Goce2011p) performs significantly better than the gradiometer-only part of the time-wise model for the entire spectrum. The official RL01 models are based on the original L1B processing, which was based on an imperfect calibration of the gravity gradients. As mainly the calibration matrix was assumed to be time invariant, drifts in some of the calibration parameters were neglected. Because this alternative approach already included a drift for the calibration, the effect of the imperfect calibration was already pointed out by that study. After an update of the L1B gravity gradient processor (see Stummer et al., 2011, 2012, Stummer, 2013), reprocessed gravity gradients were derived as official L1B gravity gradients.

Reprocessing the same data with the 'classical' time-wise approach leads to the solution EGM_TIM_-RL01repro (Brockmann et al., 2012). Comparing, the solutions again (unfortunately there are currently no original gradiometer-only solutions available), Fig. 18 shows that afterwards ITSG-Goce2011p and EGM_TIM_RL01repro are in the same accuracy level (for that part where the solution is solely determined by gradiometry). A more detailed and more sophisticated validation would be required in order to study which solution is better. That would be outside the scope of this document.

3.2.3 Making use of the alternative method

The old study has demonstrated that the general idea works well and produces an independent and at least comparable solution compared for instance to the time-wise approach. As the alternative method starts earlier in the processing chain, i.e. directly uses the measured accelerations for gravity field recovery, the alternative approach is more flexible. Following the general ideas of the time-wise processing method and the philosophy behind, this alternative approach agrees very well

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Figure 17: Gradiometer only solution computed with the alternative acceleration based approach. For all frequencies it is superior and more accurate the the gradiometer-only solution of the EGM_TIM_RL01 solution.

- It avoids transformation of observations, it even starts at an 'earlier' product closer to the original observations
- The method produces gravity field solution from GOCE observations only
- A realistic stochastic model for the accelerations can be found, thus a realistic covariance matrix for the model can be determined.

So this alternative method follows the same philosophy and agrees with the time-wise approach. The proposal is to pick up this method again and extend it such that it is able to deal with the entire mission data set. Currently only the RL01 data set was completely analyzed. Since then, the implementation was not used anymore. The basic idea is to use the alternative for different purposes:

- Validate the 'classical' processing as introduced in Sect. 3.1 performing a totally independent solution.
- Identifying the suspicious data may be easier on the acceleration level (less preprocessing in L1B generation algorithms, EGG_NOM).
- Validate the official calibration parameters, which directly follow from the algorithm. Feedback if there might be potential in the L1B product.
- A validation of the official L1B product is possible.
- Discover remaining potential in any of the processing steps of the 'classical' processing.
- Due to the higher flexibility a better final gravity field solution may be achieved.

In the other way around, the analysis performed within the line of the classical time-wise processing will help the alternative approach

- So far only stationary covariance information for the accelerations are forseen. The study and methods, for instance the methodology of time-variable decorrelation filters can be transferred to the accelerations.
- The classical time-wise processing delivers good Taylor approximation points.
- Maybe the information on the suspicious data identified with the approaches presented in Sect. 3.1 can be used to decide when new calibration parameters should be determined. This possibly reduces the suspicious data.

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Figure 18: Gradiometer only solution computed with the alternative acceleration based approach. For the frequencies determined by gradiometry it is superior and more accurate the EGM_TIM_RL01 solution. It is of the same quality then the reprocessed EGM_TIM_RL01repro solution.

A lot of studies are required to apply the method for the entire time-series, but as shown by the first study example in Sect. 3.2.2, the method is very well suited to highlight possible processing deficiencies. Although it seems that the final solution is on the same level as the classical time-wise solution, note that it was tested for the first 71 days of data only, which is the 'cleanest' segment of the entire mission data set. For the later data affected by suspicious effects, outliers and the rougher environment, further improvements are expected.

4 Improving the regularization

At this stage, for a final release of the official models, it is proposed to release gravity field solutions with different kinds of regularizations applied. The following strategies are possible, to generate a family of models which may be better regularized than the EGM_TIM_RL05 model:

- A completely unregularized solution and covariance matrix, it may be used in different applications, especially for global and local combination models.
- A simple regularization applying Kaula's rule for high degrees and the near zonal coefficients. The generation of EGM_TIM_RL05 has shown, that choosing the weight of the zonal regularization is not easy and has to be empirically found.
- A dedicated polar gap regularization, e.g. strictly inserting pseudo observation in the poles ()zero mean, adopted variances) or spatially limited other data sets. This should be an addon to a GOCE-only version. The idea would be to constrain the local impact of the regularization strictly to the poles. Test studies are required to evaluate the performance.
- Spectral methods can be used (e.g. eigen vectors) to accsess the properties of the normal equations, to identify the weakly determined base-functions.

5 Combined model

Implementing the individual updates introduced in the Sections above directly leads to an updated combined gravity field model. Improving the stochastic modelling of the gravity gradients (or accelerations) directly allows to better estimate the relative weights for segments, gradient components and observation types (SGG, SST or regularization) via variance component estimation. This section should be used to quantify the improvements of a combined model, implementing some of the updates as proposed in Sect. 2 and 3.1 as this first study was already done in Brockmann et al. (2015b).

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(a) Zoom to the low degrees.

(b) Zoom to the high degrees.

Figure 19: Comparison of EGM_TIM_RL05 and EGM_TIM_RL05.1 in terms of degree variances to the combined EGM2008 models. Shown are differences (solid) and formal errors (dashed) for both models and the RL05.1 contributers SST and SGG.

5.1 Combining an updated SST solution and a SGG solution with robustly estimated decorrelation filters

Two preliminary updated solutions were already studied in Brockmann et al. (2015b) and Zehentner (2016). The individual solution introduced in Sect. 2 and 3.1.1.2 were already combined to try to study and quantify the gain for the global solution (named EGM_TIM_RL05.1 in the upcoming figures). Fig. 19 shows the combined solution, combining and SGG normal equations (robust decorrelation filters cf. Sect. 3.1.1.2) with empirically down weighted ($w = 0.5$) SST normal equations SST (cf. Sect. 2) in terms of degree variances for the higher and lower degrees compared to EGM2008. Strong zonal Kaula regularization was applied and the high degrees were Kaula regularized with a weight derived by variance component estimation.

Fig. 19(a) highlights the problem of perfectly combining SST and gradiometry at the low degrees. Due to the remaining systematic effects (which were lowered but still are present in the current processing) the accuracy of the SST solution is too optimistic for the low degrees, where it is realistic for the high degrees (there is a tilt in the formal errors). Using weight $w_{\mathrm{SST}} \approx 1.0$ (variance component estimation result), yields a model where degrees 2–20 improve, but the gradiometry is degraded by the too optimistic SST in degrees 20–70. Using the chosen weight of 0.5, SST positive contributes for degrees 2–70, but gradiometry slightly degrades the degrees 2–18. Thus, compared to EGM_TIM_RL05, the improvements of the EGM_TIM_RL05.1_SST solution are not visible in terms of degree variances for the combined solution. Fig. 19(b) shows, that the green curve is always below the red curve (for degrees 220–280 where EGM2008 is of comparable accuracy level or more accurate then GOCE models) which clearly shows the updated EGM_TIM_RL05.1 improved compared to EGM_TIM_RL05.

To quantify the improvement, comparisons in terms of functionals of the spherical harmonic coefficients are better suited. For that purpose, geoid height differences to EGM2008 up to degree 200 of three models are shown in Fig. 20. At that resolution it is obvious, and supported by Table 3, which shows the statistics over a smooth area in the south pacific that for EGM_TIM_RL05.1, the improved solution is globally less noisy, the 'magnetic equator artifact' is reduced and the systematically larger effect around the magnetic south pole is reduced.

Table 3 shows the statistics for degrees 180, 200, 250 and 280. The RMS values are computed for the

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(a) Differences for EGM_DIR_RL05 global and local zoom to south pacific.

(b) Differences for EGM_TIM_RL05 global and local zoom to south pacific.

(c) Differences for EGM_TIM_RL05.1 global and local zoom to south pacific.

Figure 20: Geoid height differences of the test-model to EGM2008 at degree 200 (m).

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Table 3: Statistics on the difference to EGM2008 for the three test models for an area in the south pacific where the mean dynamic topography is smooth and thus EGM2008 is assumed to be very accurate. Note that not all values are computed. RMS is the root mean square error and VCM the mean accuracy of the model as computed from variance propagation for that area.

RL	RMS cm	VCM cm	RMS mGal	VCM mGal
@ d/o 180				
EGM_DIR_RL05	1.89	–	0.45	–
EGM_TIM_RL05	1.72	1.49	0.42	0.37
EGM_TIM_RL05.1	1.69	1.32	0.41	–
@ d/o 200				
EGM_DIR_RL05	2.81	–	0.76	–
EGM_TIM_RL05	2.78	2.70	0.76	0.76
EGM_TIM_RL05.1	2.58	2.40	0.71	0.67
@ d/o 250				
EGM_DIR_RL05	8.67	–	–	–
EGM_TIM_RL05	8.89	10.74	–	3.82
EGM_TIM_RL05.1	8.29	9.91	–	–
@ d/o 280				
EGM_DIR_RL05	12.46	–	–	–
EGM_TIM_RL05	12.13	17.95	–	7.00
EGM_TIM_RL05.1	11.94	16.93	–	–

shown area in the south pacific where the mean dynamic topography is smooth and thus EGM2008 is assumed to be very accurate. The given standard deviation (VCM) result from the propagation of the full covariance matrix to geoid height errors on that area. Provided is the mean value. For all test models, the updated EGM_TIM_RL05.1 shows the lowest rms value and the lowest formal error. From the numbers the gain from EGM_TIM_RL05 to EGM_TIM_RL05.1 can be summarized (based on that test area) for d/o 200 to a reduction of rms by around 7% and 11% for the formal errors. For degree 250 it is 7% for the rms and 8% for the formal error. For degree 280 it is 2% for the rms and 5% for the formal error. The propagated geoid standard deviations for degree and order 200 are shown in Fig, 21. Due to the detection and elimination of the 'suspicious' data around the magnetic poles of the robust filter estimation procedure, EGM_TIM_RL05.1 contains less data around the magnetic south pole. The covariance matrix nicely reflects that there are problems in th model, it is less accurate there.

The gain in accuracy of the updated combined model is confirmed by an external validation with GPS/Leveling data. Fig. 22 shows the validation with the Germany data set, the updated solution performs better for the spherical harmonic degrees 150 to 260. The slightly worse performance of degrees 270 and 280 is not yet understood. Either this is a random effect, related to the regularization or the higher resolution of EGM_TIM_RL05.1. The improved quality is also confirmed via the external validation procedures.

To conclude the study done in Brockmann et al. (2015b), combining a model with the updated SST processing and using the robustly estimated decorrelation filters, systematic effects in the order of centimeters are reduced (magnetic equator and magnetic poles). A global mean reduction of the error is expected to be around 5–15% depending on the area and the resolution (spherical harmonic degree). This global reduction is mainly due to the better relative weighting such that strengths of individual components/segments are better kept in combination.

5.2 Finally combined updated model

Implementing the changes discussed in the Sections before, a larger improvement is expected than discussed in Sect. 5.1. But it can not be quantified at this stage, as the combination mainly depends

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(a) For EGM_TIM_RL05. (b) For EGM_TIM_RL05.

Figure 21: Propagated geoid errors (cm) of both models at degree 200.

Figure 22: GPS/Leveling validation of the two solutions EGM_TIM_RL05 and EGM_TIM_RL05.1 with the Germany data set. Courtesy: Thomas Gruber, IAPG, TU München

on the results of the processing proposed in Sect. 2 and 3. It will be very interesting, how the gradiometry solutions improves using the time variable filters and how the alternative acceleration based processing will perform in a combination. But the mean improvement around 10% is the least which is expected.

6 Possible product extensions and updates to improve usage and usability of the models and the covariance matrices

To increase the usage of the products, we suggest to distribute the new products in addition to the official channels via an easy accessible ftp server at the institutes. There, additional and simplified formats especially for the covariance matrices or normal equations can be provided. We suggest to put individual normal equations for SST, SGG and regularisation there. Together with the normals, we would provided some easy-to-use readers, directly reading the matrices into the memory of a shared memory computer equipped with enough memory. These readers might be MatLab or Python readers. In addition, for instance reordering routines can be provided, reordering the parameters from for instance from a degree-wise ordering to an order-wise ordering. If suitable, maybe sparse approximations can be created (block-diagonal, Kite, flexible Kite and so on). If that is possible and useful depends on the structure of the final normal equations (Brockmann and Schuh, 2011).

7 Summary and Conclusions

This documented collects a summery on performed studies that were made by WP6000 to find out if there is some room left to improve the final GOCE-only gravity field model. In addition, updates of the processing are proposed to improve the final product derived from GOCE observations, a global gravity field model based on SST and SGG observations and its corresponding full covariance matrix. The basic philosophy of the time-wise processing approach should be kept, i.e. producing a gravity field model purely based on the observations of the GOCE satellite. Thus, the model describes the gravity field as seen by GOCE and its uncertainties.

7.1 Short summary

The analysis of the two major data types the gravity field is estimated from – the satellites orbits and the gravity gradients – were separately performed. Detailed studies were performed which indicate that from both data types improvements for the gravity field can be expected, if the analysis procedures are refined. For both data types, the proposed updates are related to the handling of suspicious data in the time-series. These updates are required to improve the former release by a few additional percentages. For both data types separately, improvements for the processing strategies were identified, which yield to better individual gravity field solutions (SST- and SGG-only). Consequently, the finally combined solution will improve, inheriting the improvements of the individual models.

Currently, for the SST part, the improvements are mainly due to better processing and pre-processing of 'suspicious data' occurring in the GPS phase observations around the magnetic equator. These effects, hardly visible in the kinematic orbits, lead to systematic errors in the SST-based long-wavelength gravity field model in the range of ± 2 cm along the magnetic equator. Applying the updates discussed in Sect. 2, requiring a re-computation of the kinematic orbits according to Zehentner (2016), the systematic effect can be reduced to at least below ± 1 cm close to the magnetic equator. Consequently, a huge reduction of the systematic errors is possible, reducing the systematic errors about a factor of 2 (Brockmann et al., 2015b).

For the SGG part, all starting points are related to an improved time-variable modelling of the stochastic characteristics and a better modeling of suspicious data sequences. The proposed improvements, all related to the estimation of the decorrelation filters, yield a better relative weighting of the different segments of the time series and the different tensor components, resulting in a better SGG-only model from all data segments. In addition to that, systematic errors/differences for instance around the magnetic south pole are reduced.

Implementing all updates discussed above, a combined model with a mean error reduction around 10 % is expected. In addition to that, systematic errors are reduced, resulting in an reduction at centimeter level in the affected regions. The provided covariance matrix is expected to be even more realistic, reflecting also geographically the problematic areas where increased noise in the gravity gradients is observed.

It is proposed to verify the time-wise processing with an alternative approach, using directly accelerations to estimate the gravity field. The approach co-estimates the calibration of the gravity gradients, resulting in an independent verification of the L1B gravity gradient calibration. Due to the design of the approach, the calibration parameters can be estimated in an arbitrary temporal resolution. To account for the changes in between, an arbitrary deterministic model can be used, for instance a linear drift. The results will be used to derive an independent gravity field model, expected to be at least on the same accuracy level then the improved model discussed above. As it is more flexible, and the suspicious data might be better handled on acceleration level (as the differentiation is not performed), even a better model is expected. Having both approaches implemented and running in parallel, they can be validated against each other, and used to identify open issues in any of the processing steps from accelerations to the global Earth's gravity field.

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7.2 Short conclusions

The WP6000 project team strongly supports the idea of reprocessing the entire mission data set for the SST data and the SGG data. As the results will definitely improve, the best possible solutions should become official products. Support by ESA is requested, as some of the proposed studies and updates will require detailed scientific studies, implementing and refining methods not used in the context of global gravity field recovery and GOCE data analysis before.

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