Analysis and Refinement for an Integrated Approach Estimating the Ocean's MDT from Altimetry and GOCE

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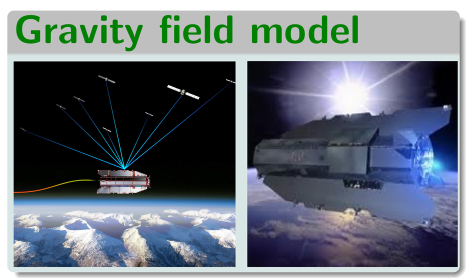
Abstract

The ocean's dynamic topography (DT) as the difference between the mean sea surface and the geoid reflects many characteristics of the general ocean circulation. Consequently, it provides valuable information for evaluating or tuning ocean circulation models.
Along track altimetry data, measuring the sea surface height and satellite-based gravity field information are used to estimate a local model for the dynamic topography and the corresponding uncertainty information. For that purpose, an integrated approach was developed to estimate the dynamic topography and the gravity field in a rigorous adjustment. Complementary observation types, in this study altimetric sea surface height measurements and GOCE gravity field observables are combined to estimate the gravity field, a local mean dynamic topography model (MDT) and its full covariance matrix.
Within this study we present an analysis of the altimetry residuals over the North Atlantic Ocean resulting from the joint least-squares adjustment. Different set-ups for the base functions are analyzed. In addition different stochastic models for the along track altimetry observations (covariance functions and decorrelation filters) and their impact on the target parameters, their accuracy estimates as well as the altimetry residuals are assessed.

Background and Motivation for the Study

Estimation of the Ocean's Dynamic Topography from along track altimetry

Gravity field model

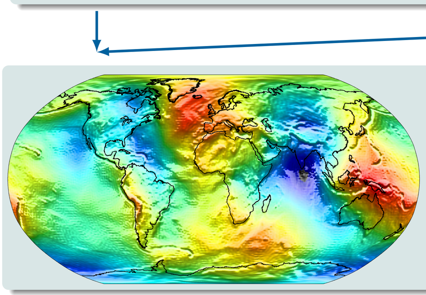


- ⋆ GOCE-based
- ⋆ several missions
- ⋆ band-limited
- ⋆ as full NEQs

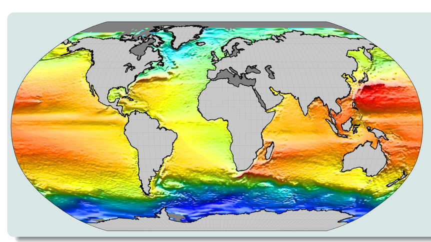
Altimetry

- ⋆ along-track SSH
- ⋆ high resolution
- ⋆ e.g. AVISO CorSSH product

separation



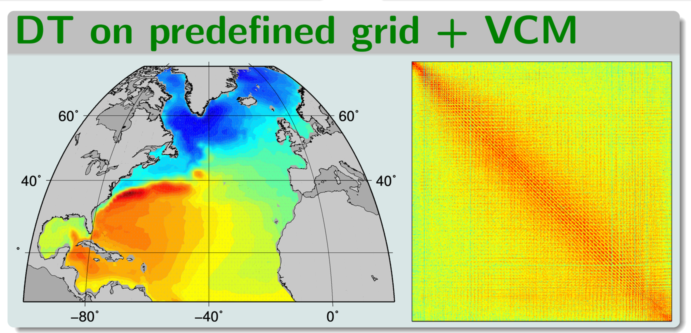
Geoid
- ⋆ global spherical harmonics
- ⋆ high resolution



Dyn. Topography
- ⋆ finite element base functions
- ⋆ local area

spherical harmonics:
- ⋆ currently: buffer for high frequency altimetry
- ⋆ eliminated from NEQs
- ⋆ future: target quantity

DT on predefined grid + VCM



finite elements:
- ⋆ target quantity
- ⋆ currently: local focus
- ⋆ resolution defined by grid & base functions

Methods, algorithms & implementation developed and used in Becker (2012), Müller (2014), Müller et al. (2015), Becker et al. (2014a,b), Brockmann (2014), Brockmann et al. (2014)

Focus of the Study: Description & Modeling of Uncertainties

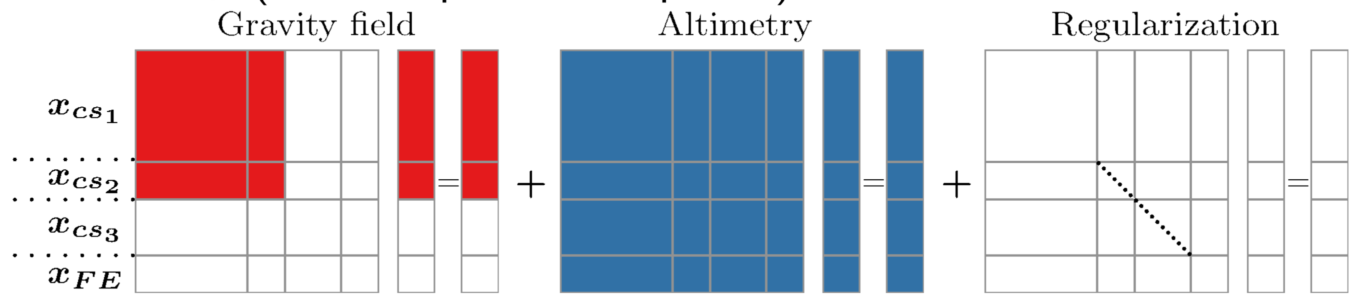
Modeling the uncertainties of involved observation groups

Gravity Field
- use of unconstrained models and its full covariance matrix: $\mathbf{x}_{gf}$ and $\mathbf{\Sigma}_{\mathbf{x}_{gf}\mathbf{x}_{gf}}$
- NEQs as sufficient statistics of original observations can be determined (inv.)
- global gravity field models with realistic covariance information are available

Altimetric Observations
- measurements: time series along the orbit
- model describes: stochastic measurement errors, model errors (omission)
- stationarity is assumed
- idea: iterative process to refine the stochastic model by SSH residual analysis

Setup of an iterative procedure:

1. Input data and information
   - predefined finite element setup (triangulation and base function type)
   - predefined spherical harmonic setup (maximal resolution)
   - NEQs of gravity field model: $\mathbf{N}_{gf}$, $\mathbf{n}_{gf}$,
   - OEQs of ssh observation of a mission i: $\mathbf{A}_{ssh_i}$, $\boldsymbol{\ell}_{ssh_i}$, and initial covariance model $\mathbf{Q}^{(0)}_{ssh_i}$
   - initial weights for all observation groups (w_i and w_{gf})
2. Compute NEQs for ssh observations, $\mathbf{N}_{ssh_i} = \mathbf{A}^T_{ssh_i}\mathbf{Q}^{(0)-1}_{ssh_i}\mathbf{A}_{ssh_i}$, $\mathbf{n}_{ssh_i} = \mathbf{A}^T_{ssh_i}\mathbf{Q}^{(0)-1}_{ssh_i}\mathbf{A}_{ssh_i}$,
3. Combine NEQs (different parameter spaces!)



$$\mathbf{N} = w_{gf}\mathbf{N}_{gf} + \sum_i w_i \mathbf{N}_{ssh_i}, \qquad \mathbf{n} = w_{gf}\mathbf{n}_{gf} + \sum_i w_i \mathbf{n}_{ssh_i} \qquad (1)$$

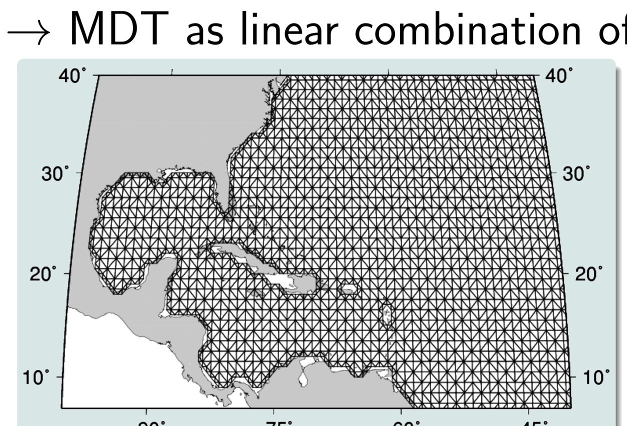
4. Rigorous parameter estimation (huge dimensional) $\tilde{\mathbf{x}} = \mathbf{N}^{-1}\mathbf{n}$
5. Estimate weights for all observation groups (w_i and w_{gf}), continue with 3.
6. Compute SSH residuals $\mathbf{v}_{ssh_i} = \mathbf{A}_{ssh_i}\tilde{\mathbf{x}} - \boldsymbol{\ell}_{ssh_i}$
7. Analyse $\mathbf{v}_{ssh_i}$ and adjust a stochastic model to $\mathbf{v}_{ssh_i}$, update $\mathbf{Q}^{(1)}_{ssh_i}$ and continue with 2.

Parameterization used in this Study

Gravity Field
- spherical harmonic base functions, physically motivated
- global support
- in general up to degree 720 possible
- in this specific study: d/o 360 and 540

Altimetric Observations
- finite element base functions (linear, quadratic and Argyris elements)
- predefined triangulation, local support
- here: linear elements, triangulated $1^\circ \times 1^\circ$ grid (IFEOM), constant in time

here: linear finite elements are set up for an local area (North Atlantic), constant in time → MDT as linear combination of base functions



$$\mathrm{MDT}(\theta,\lambda) = \sum_{k \in K} a_k b_k(\theta,\lambda), \qquad \mathbf{x}_{FE} = [a_k]$$

- nodal points predefined by ocean model
- unknowns a_k are the MDT at the nodal points
- **direct access to target quantities**

Stochastic Models to Describe Uncertainties

Error covariance functions in time are used to describe the uncertainties of the along-track SSH observations, they are derived by two approaches

1. Initial model: sum of exp functions adjusted to residuals wrt. ref. models (MDT, geoid)

$$\rho_i(\Delta t) = a_i e^{-b_i * \Delta t} + c_i * e^{-d_i * \Delta t}$$

a_i, b_i, c_i and d_i mission specific cf. Becker (2012)

2. Iterative refinement: robust estimation of an AR-process from the residuals $\mathcal{V}_{ssh_i}$

$$\mathcal{V}_{i,k} = \mathcal{S}_{i,k} + \mathcal{N}_{i,k}$$
$$\mathcal{S}_k = \sum_{j=1}^{p} \alpha_j \mathcal{S}_{k-j} + \mathcal{E}_k$$

$\mathcal{S}_k$ ... stochastic process in AR representation
$\mathcal{E}_k$ ... innovation process
$\mathcal{N}_k$ ... additive process (noise)
cf. Koch et al. (2010), Schuh et al. (2015)

- AR-process (order 16) coefficients are computed per mission and per cycle
- compute correlation function per cycle from coefficients $\alpha_{i,j}$
- a median correlation function is determined per mission from all cycles
- median correlation function is analytically approximated by finite exp & cos function

$$\rho_i(\Delta t) = (1 - a_i) + a_i \cos(b_i \Delta t/\pi)\, e^{-c_i \Delta t}\mathrm{CovFunFin2D}(d_i)$$

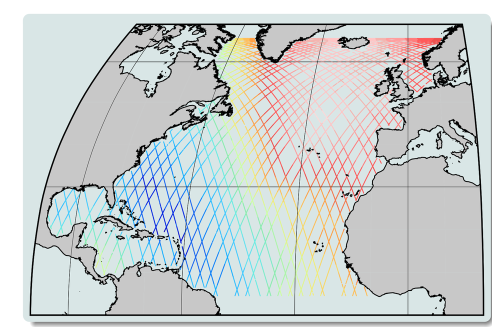
Data used in this Test Study

EGM_TIM_RL05 NEQs

i	l_{min}	l_{max}
SGG	2	281
SST	2	53–64
KAULA	181	360/540

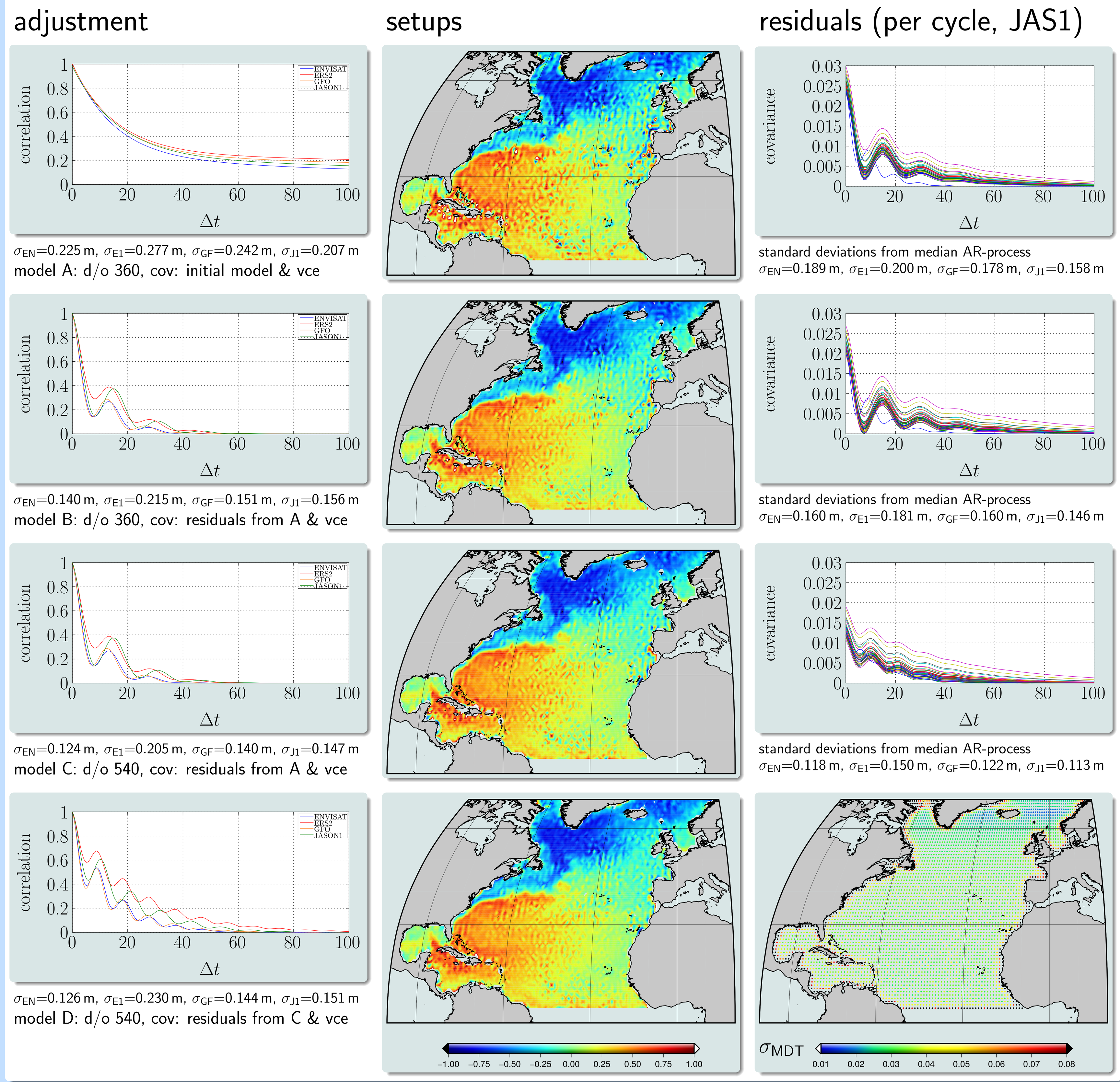
SSH data from 4 missions, data of 2007

i	# cycle	cycles	# obs
ERS2	1	121	157 512
ENVISAT	12	53–64	1 972 985
GFO	25	184–208	1 724 166
JASON1	44	179–222	2 747 694



Results

Correlation functions used in adjustment | MDT solutions for different setups | Covariance functions from residuals (per cycle, JAS1)



σ_{EN}=0.225 m, σ_{E1}=0.277 m, σ_{GF}=0.242 m, σ_{J1}=0.207 m
model A: d/o 360, cov: initial model & vce

standard deviations from median AR-process
σ_{EN}=0.189 m, σ_{E1}=0.200 m, σ_{GF}=0.178 m, σ_{J1}=0.158 m

σ_{EN}=0.140 m, σ_{E1}=0.215 m, σ_{GF}=0.151 m, σ_{J1}=0.156 m
model B: d/o 360, cov: residuals from A & vce

standard deviations from median AR-process
σ_{EN}=0.160 m, σ_{E1}=0.181 m, σ_{GF}=0.160 m, σ_{J1}=0.146 m

σ_{EN}=0.124 m, σ_{E1}=0.205 m, σ_{GF}=0.140 m, σ_{J1}=0.147 m
model C: d/o 540, cov: residuals from A & vce

standard deviations from median AR-process
σ_{EN}=0.118 m, σ_{E1}=0.150 m, σ_{GF}=0.122 m, σ_{J1}=0.113 m

σ_{EN}=0.126 m, σ_{E1}=0.230 m, σ_{GF}=0.144 m, σ_{J1}=0.151 m
model D: d/o 540, cov: residuals from C & vce

Summary, Conclusions and Outlook

- MDT estimation from satellite data only (along-track altimetry and GOCE geoid model) including end-to-end covariance modeling ⇒ HPC required
- Advanced covariance modeling of ssh observations has positive effect on MDT solutions
- Covariance modeling via AR-processes converges, models observation and model errors
- Covariance model changes if model changes (here spherical harmonic resolution)
- Remaining inconsistencies: variances of AR-processes vs. estimated variance components
- Direct use of individual AR-filters for decorrelation
- Study consequences if MDT parameterization is changed (e.g. quadratic finite elements)
- Study consequences if DT parameterized (time-variable finite elements)

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