A parametric representation to integrate current observations into the estimation of the DOT

Jan Martin Brockmann[1], Christian Neyers[1], Luciana Fenoglio[2] and Wolf-Dieter Schuh[1]
Institute of Geodesy and Geoinformation, [1]Theoretical Geodesy Group, [2]APMG Group, University of Bonn

Abstract

The geodetic estimation of the mean dynamic ocean topography (MDT) as the difference between the sea surface height (SSH) and the geoid remains, despite the simple relation, still a difficult task. Mainly, the spectral inconsistency between the available altimetric SSH observations and the geoid causes problems in the separation process. This is complemented by the accuracy characteristics of the satellite derived geoid information, as it is only sufficiently accurate for a resolution of about 100 km. As soon as further observations are integrated, which can be linked only to the MDT, the separation process is strongly supported. One type of information which can be used are (geostrophic) currents.
Within a national project, we study parametric approaches which represent the MDT by a mathematical function. In contrast to the commonly used grid based approaches -- which provide function values on a predefined grid -- the mathematical surface is continuously defined. With a focus on C1 -smooth finite elements, the surface as well as its first derivatives, which are related to the geostrophic currents, are smooth. Due to the parametric nature of the proposed representation, it is possible to integrate any observation which can be linked to either the geoid, or the MDT into the estimation process. Possible observation groups are geostrophic current observations derived from surface drifters or, as in focus of the current project, SAR derived radial surface velocities.

Motivation and Introduction

Typical geodetic DOT estimation: Difference of a Mean Sea Surface (MSS) model and a geoid (N) and then: some filtering (e.g. [1, 6])

$$\zeta = \mathbf{F}(MSS - N) \qquad (1)$$

Problems of typical approaches
- grid defined by MSS
- multi-step procedures
- MSS models lack of an error model
- the used geoid model is often not independent
- the geoid error is not accounted for
- integration of complementary data, e.g. currents, critical
- multi-mission analysis

Our approach to overcome deficits
- ⇒ continuous parametric model
- ⇒ integrated estimation: DOT + geoid
- ⇒ along-track obs. + uncertainty
- ⇒ use satellite based geoid models, independent of altimetry
- ⇒ use spherical harmonics + full VCM
- ⇒ advanced observation equations due to continuous model
- ⇒ continuous model

Parametric approach for the DOT estimation [10, 4, 5, 3]

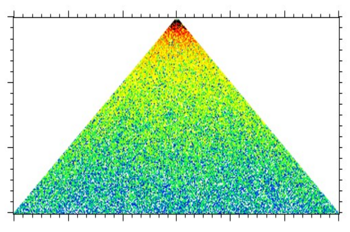
geoid/gravity
- observations: satellite-based gravity field NEQ (e.g. GOCO), smoothness
- full VCM + ommision
- spherical harmonics

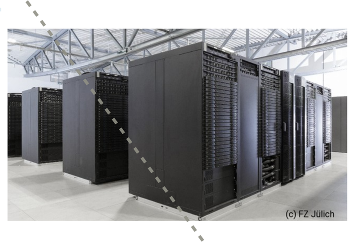
sea surface height
- observations: along-track SSH of multiple missions (e.g. altimetry, SWOT)
- empirical error model
- potential: tide gauges

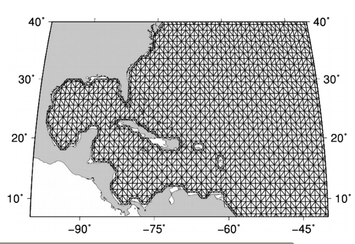
ocean surface
- observations: drifters, surface currents (e.g. **SVP, SAR, SKIM**), smoothness conditions
- empirical error model

$l_{GF} = f(x_{GF})$ $\quad l_{SSH} = f(x_{GF}, x_{DOT})$ $\quad l_{OG} = f(x_{DOT})$

joint least squares adjustment
- Estimate gravity field & DOT
- seperation of SSH
- weighted combination
- high-dimensional, HPC
- spatial & temporal filtering

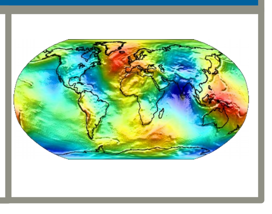




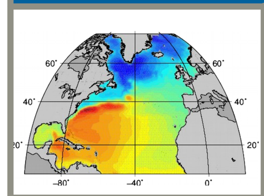


estimated parameters

gravity field
- parametric model: spherical harmonics
- parameters: $x_{GF} = [\, c_{lm}, s_{lm} \,]$
- full VCM



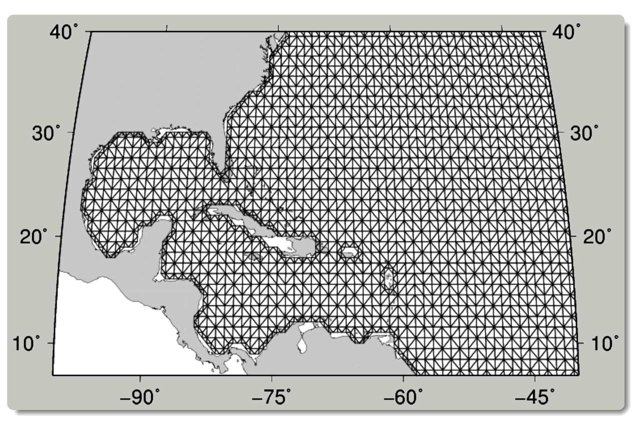
dynamic ocean topography
- parametric model: finite element space
- parameters: $x_{DOT} = [\, a_k(t) \,]$
- Full VCM



Finite elements to describe the DOT

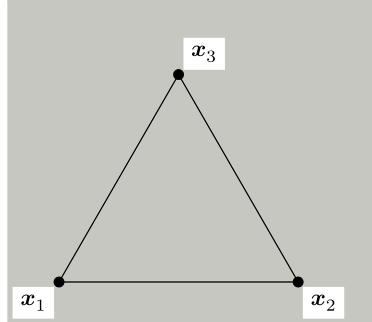
Mean DOT is represented by linear combination of finite element base functions $b_k(\theta, \lambda)$

$$\zeta(\theta, \lambda) = \sum_{k \in K} a_k b_k(\theta, \lambda), \qquad \mathbf{x}_{DOT} = [a_k] \qquad (2)$$



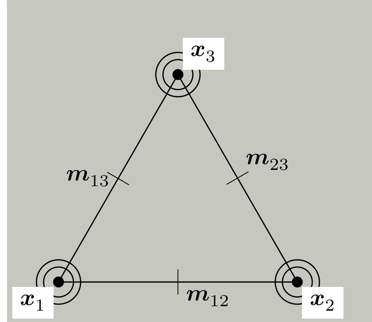
- continuous model in space (C^0/C^1-smooth)
- unknowns a_k interpretable (e.g. DOT, 1st/2nd derivative, ...)
- finite element space defines filtering/spatial resolution
- observation equations in any location and functional (values, derivatives, integrals)

C^1-smooth element: linear element



- 3 degrees of freedom
- simple: DOT in nodes
- jumps in derivatives
- details: [3]

C^1-smooth element: ARGYRIS element

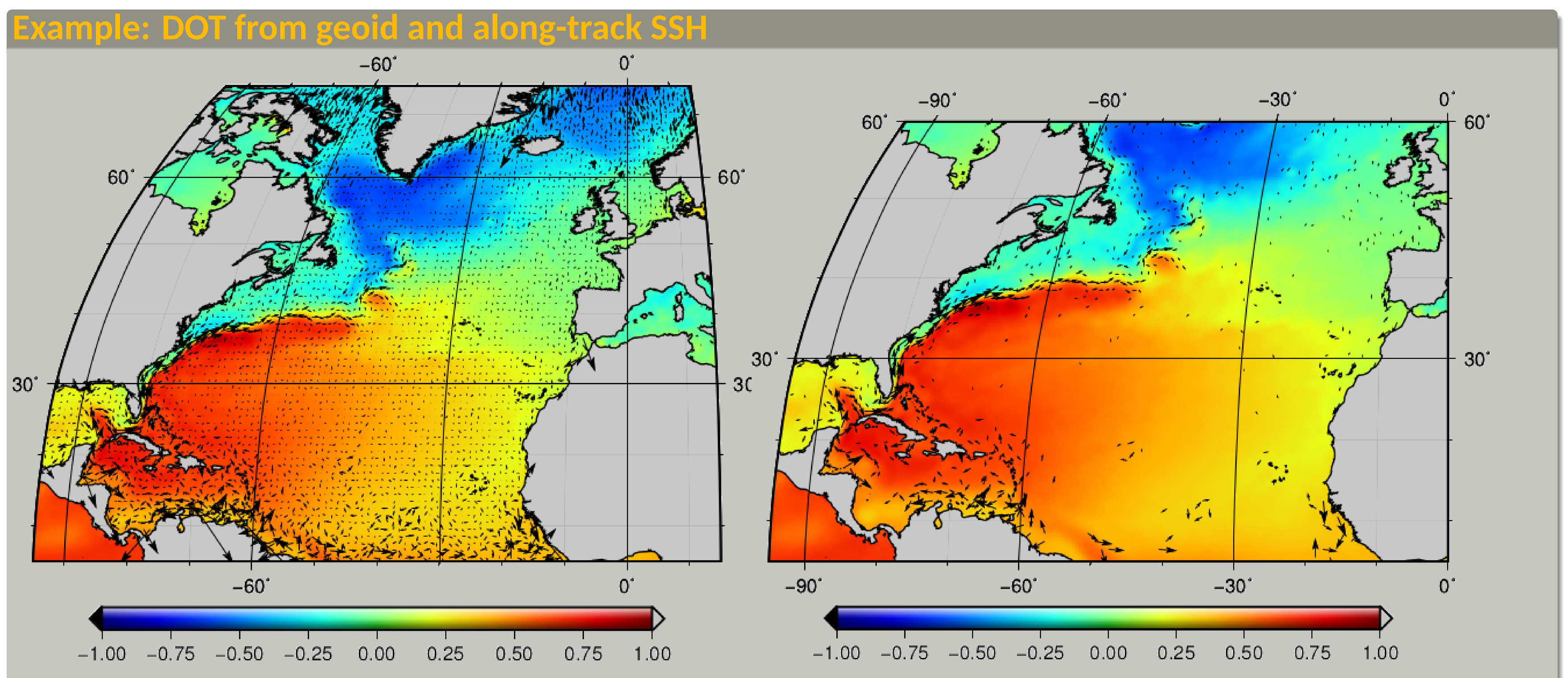


- 21 degrees of freedom
- C^1-smooth ⇒ integrate currents
- project PARASURV
- details: [2, 11]

Opportunity: Uncertainties of the DOT

If effort is spend on modeling the uncertainties of the input observations (gravity field, SSH, currents...) ⇒ rigorous propagation to DOT
⇒ rigorous propagation to derived quantities like geostrophic currents [8]

Example: DOT from geoid and along-track SSH

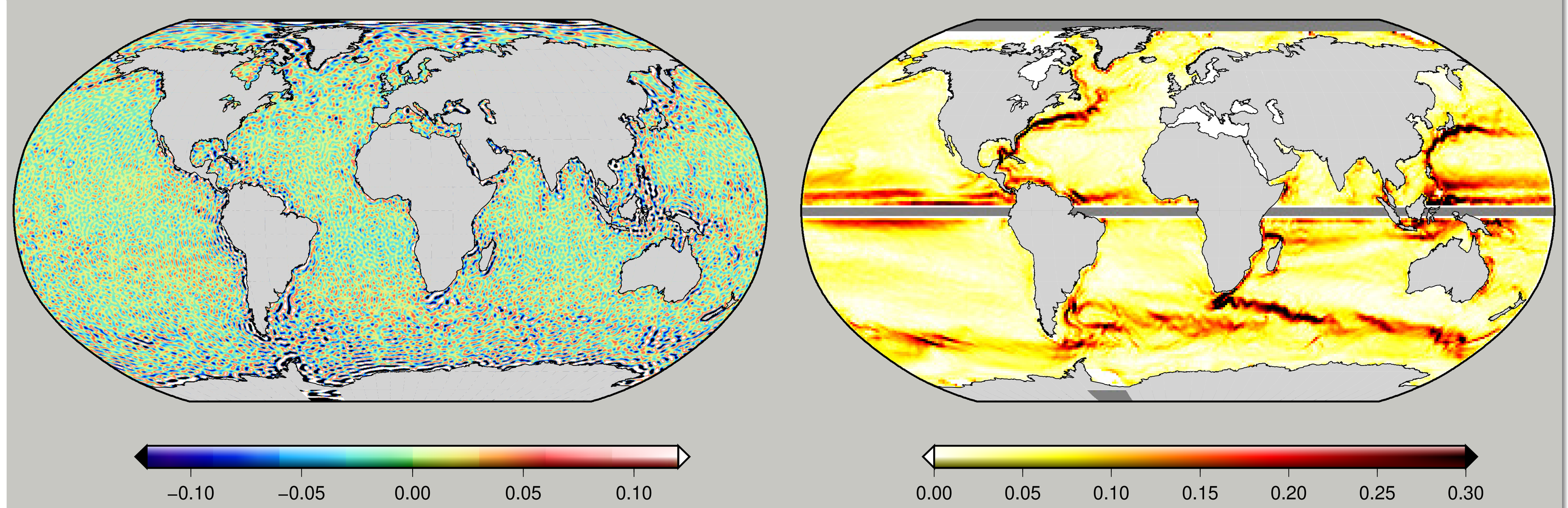


DOT model and derived currents — DOT model and significant currents

Opportunity: Updating the marine geoid

If separation is supported by observations linked to the DOT ⇒ improvement of geoid

Example: Combined global geoid contain significant ocean signal



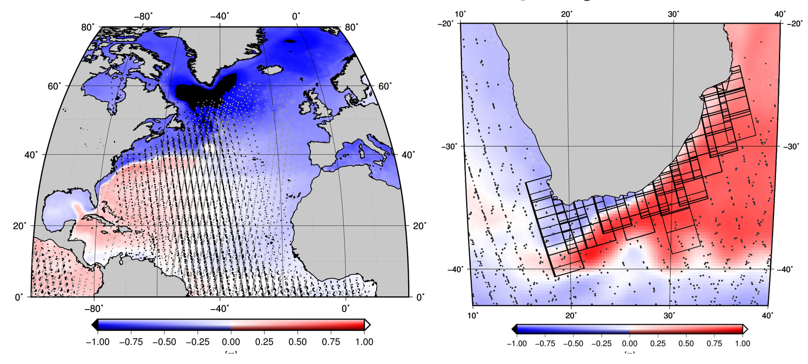
Geoid: EGM2008 - TIM_RL06 [7] — CLS09 [12] velocity amplitudes

Opportunity: Integrate surface current observations

Current available global data sets: SAR derived radial surface velocities [9]

Idea within the PARASURV project
- Motivated by Sentinel-1 OCN product
- C^1-smooth representation of DOT
- include directional derivatives as observations
- requires products to reduce ageostrophic currents
- ⇒ until now disappointing data quality, but methodological focus
- ⇒ potential SKIM mission promising

Test areas defined for the project



(a) North Atlantic Ocean (b) SW Indian Ocean
Available RVL observations (bounding boxes IW, gray dots (WV))

Summary, conclusions and recommendations

Surface current data collected by satellites strongly supports the separation of SSH

- continuous description of the DOT
- using along-track measurements allows uncertainty modeling
- propagation to currents
- ⇒ providing uncertainty for models
- requires global independent current data (e.g. SAR, SKIM)
- requires reduction of ageostrophic component (e.g. GlobCurrent)
- ⇒ observation of currents from space

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Institute of Geodesy and Geoinformation
Theoretical Geodesy Group
University of Bonn

Contact:
Jan Martin Brockmann
brockmann@geod.uni-bonn.de

Nussallee 17
D-53115 Bonn
www.igg.uni-bonn.de/tg/