Phase Unwrapping problem in differential radar interferometry (D-InSAR) analysis based on the Lower-Rhine-Embayment

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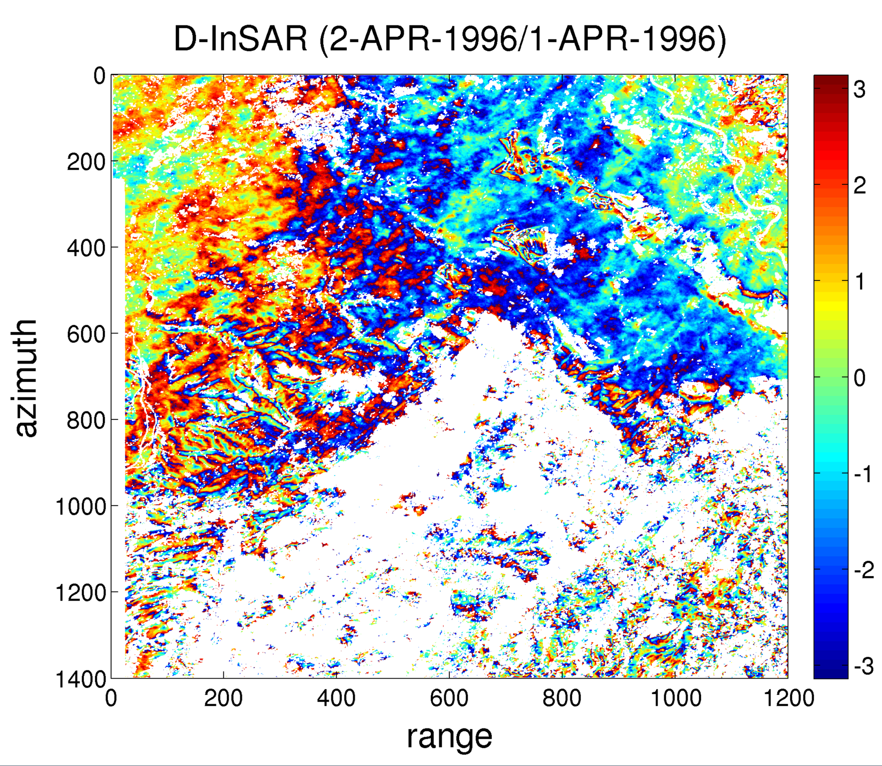
Abstract

To detect ground deformation time series in the Lower-Rhine-Embayment we use differential radar interferometry (D-InSAR) data. In more detail we use a multitemporal D-InSAR stack of 191 interferograms from the European Remote Sensing satellites ERS 1/2. Especially in areas of very rapid surface changes or deformations the 35-day repeat cycle is too long and leads to decorrelated and thus noisy data. These effects make it a challenging task to unwrap the phase. A rather popular technique to reconstruct the phase is to recast the problem into a network with nodes and arcs, searching for the Minimum Cost Flow (MCF). The Basic MCF approach only worls within one single interferogram. An extended version to the three dimensional case exploits both the spatial as well as the temporal information. Based on simulated data it is shown that this extended version improves the correctness of the unwrapped phase. For a better understanding of the solution the Phase Unwrapping problem is also compared to a L1-Norm constrained adjustment of a leveling network.

Motivation: Lower-Rhine-Embayment



Characteristics:
- Rural area
- Brown coal area
  - Open cast-mines: Garzweiler, Hambach and Inden
  - Groundwater lowering

→ Subaerial surface deformation

Aim:
- Analysis of geophysical phenomena (e.g. deformation time series)

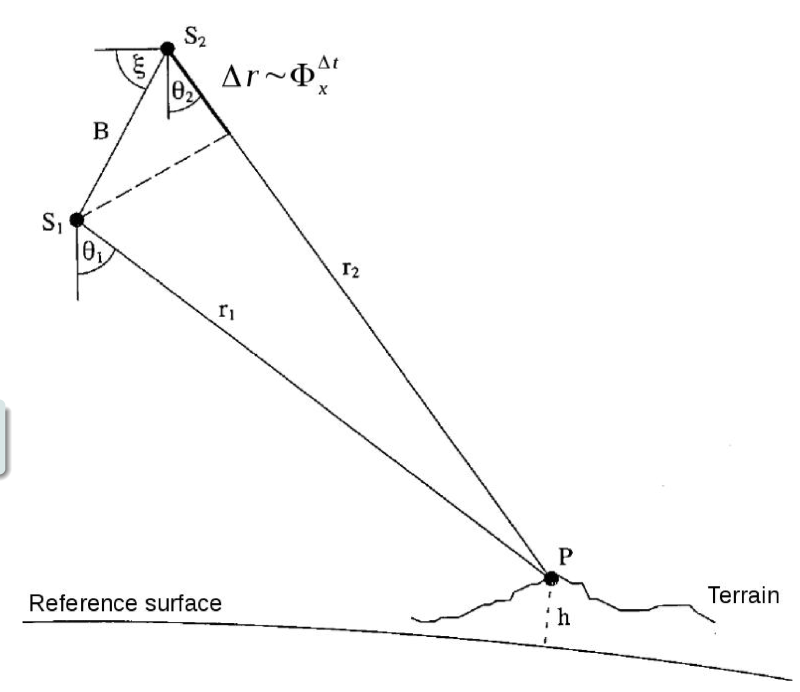
Data:
- Spaceborne radar ERS 1/2 (European Remote Sensing) data from 1992-2000

Differential Synthetic Aperture Radar-Interferometry (D-InSAR)

Principle:
- Satellite based remote sensing system
- Two aquisitions to achieve spatial representation
- Measurement is the interferometric phase $\psi_x^{\Delta t}$
- Phase is the sum of several effects:

$$\psi_x^{\Delta t} = \phi_{x,topo}^{\Delta t} + \phi_{x,defo}^{\Delta t} + \phi_{x,noise}^{\Delta t} - 2\pi k_x^{\Delta t} + \ldots$$

- To extract $\phi_{x,defo}^{\Delta t}$:
  - Reduction of the topography by subtracting a digital model
  - Only use stable scatterer to reduce noise



Schwäbisch 1995, p.14

Problem:
- Phase can only be measured modulu 2π:

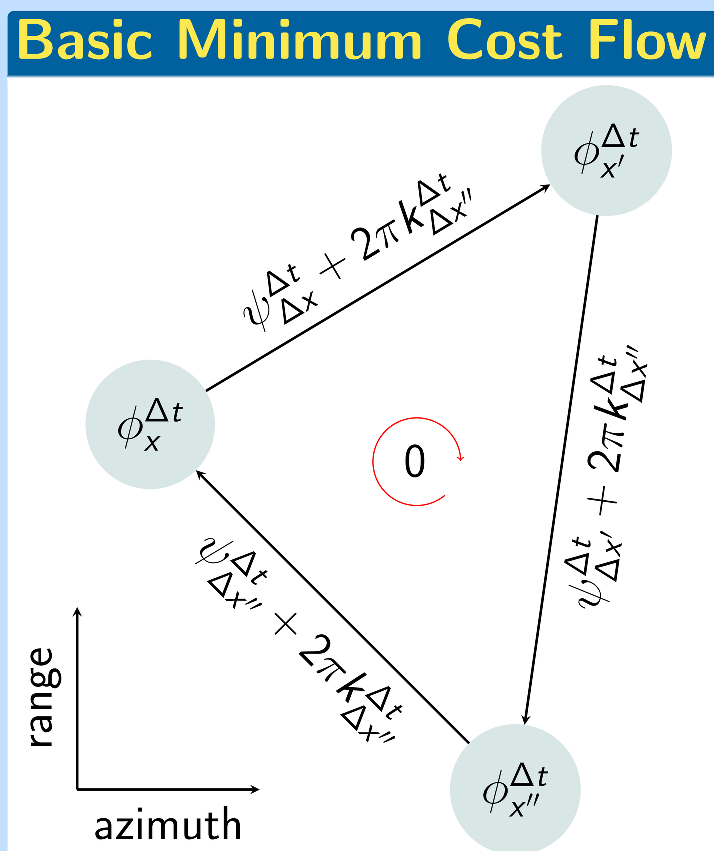
$$\phi_x^{\Delta t} = \psi_x^{\Delta t} + 2\pi k_x^{\Delta t}$$

- No unique solution for unknown phase ambiguity factor $k_x^{\Delta t}$

Phase Unwrapping (PhU) technique:
- Reconstruction with phase gradient
- Recast the problem into a network with nodes and arcs

→ Searching for the Minimum Cost Flow

Basic Minimum Cost Flow (MCF) Approach



- Basis: Single interferogram
- Network: Delaunay triangulation in the azimuth-range plane based on stable scatterer
- Unknown parameter: Phase ambiguity factor $k_{\Delta x_i}^{\Delta t}$ along each arc
- Problem formulation:

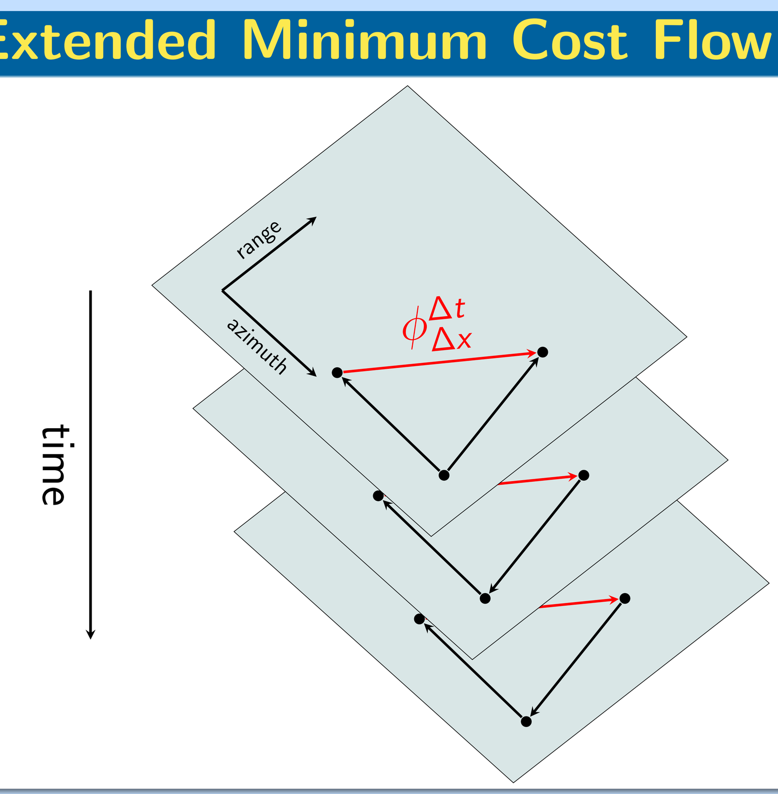
Integer Linear Program

obj. func.: $\sum_{i=1}^{n} p_{\Delta x_i}^{\Delta t}|k_{\Delta x_i}^{\Delta t}| \ldots \min$

constraints: $\mathbf{B}^T(\psi_{\Delta x}^{\Delta t} + 2\pi \mathbf{k}_{\Delta x}^{\Delta t}) = \mathbf{0}$

variable: $\mathbf{k}_{\Delta x}^{\Delta t} \in \mathbb{Z}^n$

Extended Minimum Cost Flow (EMCF) Approach



- Basis: Multitemporal D-InSAR images (stack)
- Idea: Exploitation of the temporal information to bootstrap the basic MCF approach
- 2 Networks:
  1. Spatial Delaunay triangulation
  2. Temporal Delaunay triangulation
- Workflow:
  1. Temporal PhU for each phase gradient
  2. Spatial PhU for each interferogram

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Solving the PhU Problem with the Primal-Dual Algorithm

Reformulation of the Phase Unwrapping problem

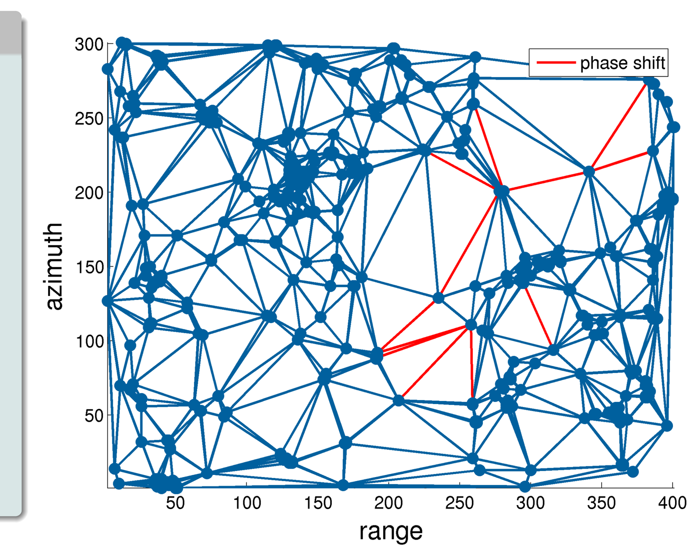
obj. func.: $\mathbf{p}_{\Delta x}^{\Delta t\,T}\mathbf{v}_{\Delta x}^{\Delta t,+} + \mathbf{p}_{\Delta x}^{\Delta t\,T}\mathbf{v}_{\Delta x}^{\Delta t,-} \ldots \min$

constraints: $\mathbf{B}^T\mathbf{v}_{\Delta x}^{\Delta t,+} - \mathbf{B}^T\mathbf{v}_{\Delta x}^{\Delta t,-} = -\underbrace{\mathbf{B}^T\psi_{\Delta x}^{\Delta t}}_{\mathbf{w}}$

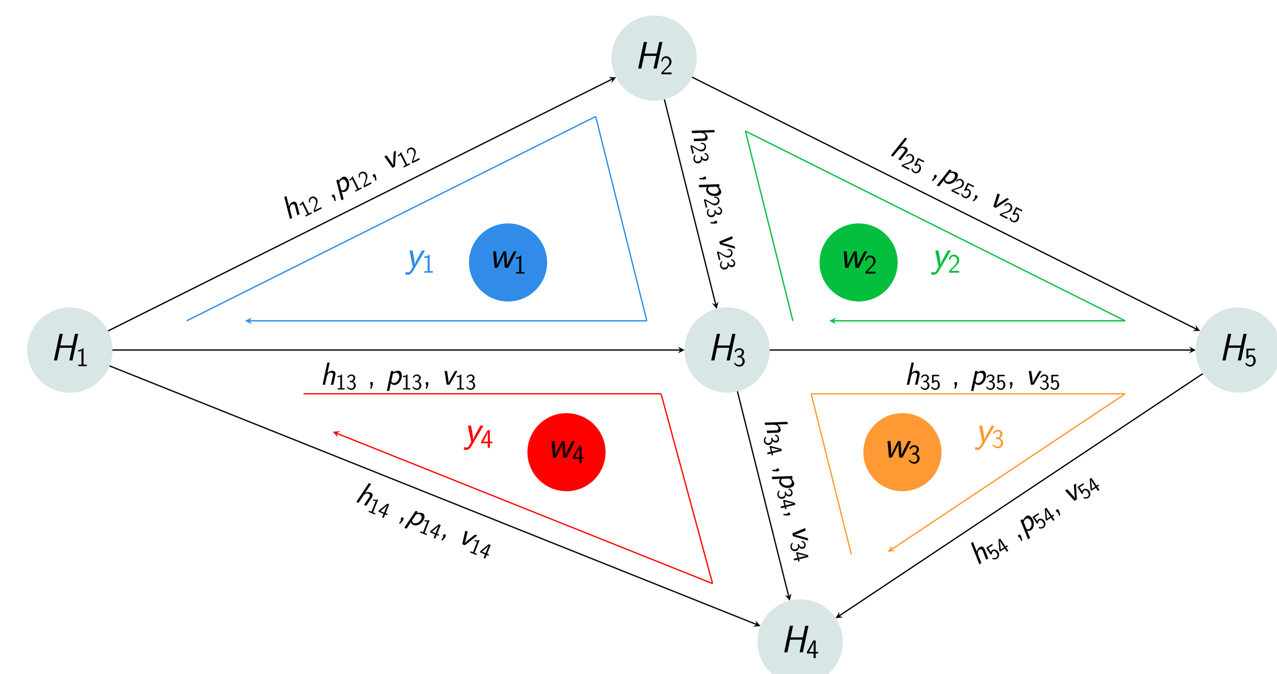
variable: $\mathbf{v}_{\Delta x}^{\Delta t,+}, \mathbf{v}_{\Delta x}^{\Delta t,-} \in \mathbb{R}^{n,\geq 0}$

⇒ phase ambiguity factor

$$\mathbf{k}_{\Delta x}^{\Delta t} = \left[\frac{\mathbf{v}_{\Delta x}^{\Delta t,+} - \mathbf{v}_{\Delta x}^{\Delta t,-}}{2\pi}\right]$$



Idea:
- Phase Unwrapping problem is equivalent to a L1-Norm constrained adjustment of a leveling network
- Leveling network transformed into a flow problem:



Linear Program (primal):

obj. func.: $\Phi(\mathbf{v}) = \mathbf{p}^T\mathbf{v}^+ + \mathbf{p}^T\mathbf{v}^- \ldots \min$

constraints: $\mathbf{B}^T\mathbf{v}^+ - \mathbf{B}^T\mathbf{v}^- = -\underbrace{\mathbf{B}^T\mathbf{h}}_{\mathbf{w}}$

variable: $\mathbf{v}^+, \mathbf{v}^- \in \mathbb{R}^{n,\geq 0}$

Linear Program (dual):

obj. func.: $\Phi(\mathbf{y}) = -\mathbf{w}^T\mathbf{y} \ldots \max$

constraints: $-\mathbf{p} \leq \mathbf{B}\mathbf{y} \leq \mathbf{p}$ (capacity bound)

variable: $\mathbf{y} \in \mathbb{R}^{n-m}$

Complementary Slackness Condition:

Strong duality

$$\Phi(\mathbf{v}) = \Phi(\mathbf{y})$$

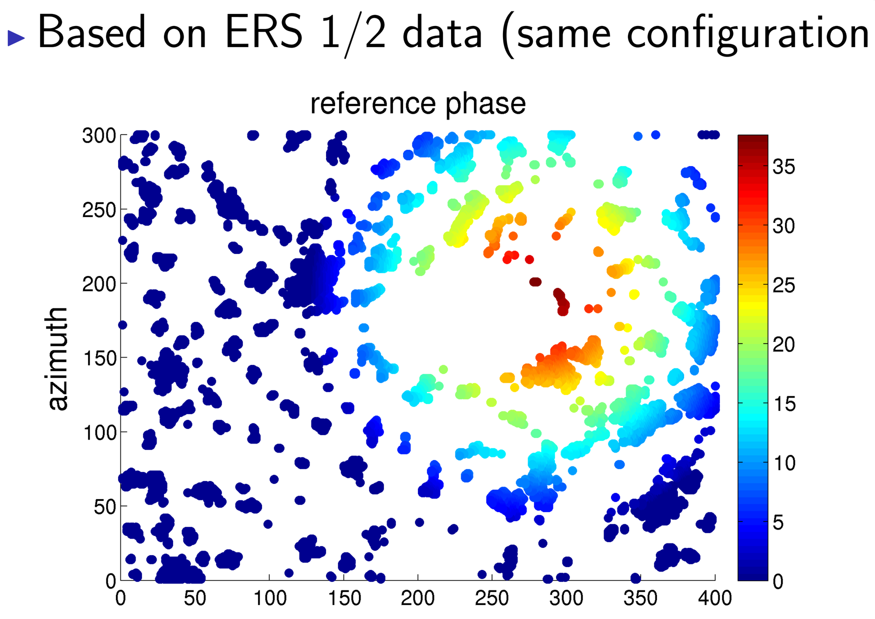
leads to

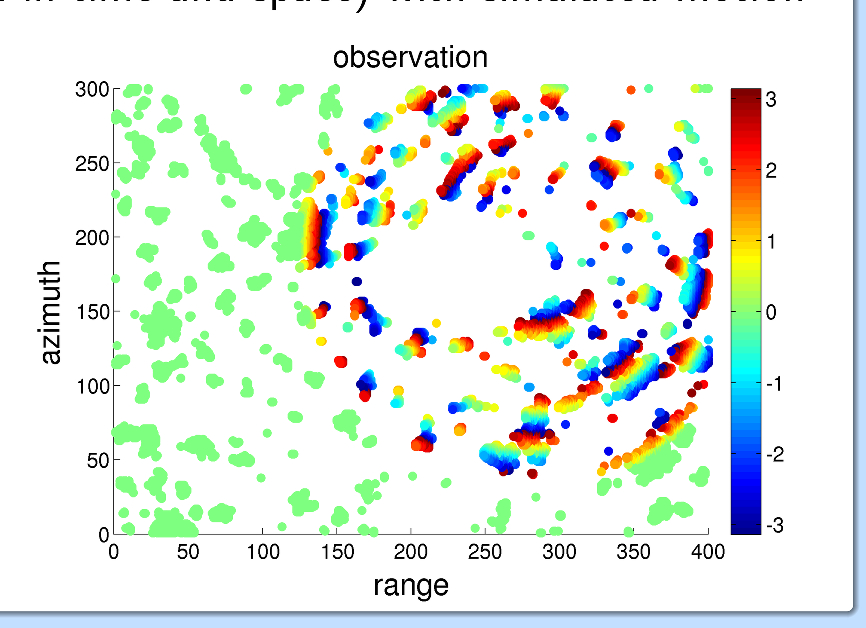
$$v_i \geq 0 \Leftrightarrow \mathbf{B}_i y_i = -p_i$$
$$v_i \leq 0 \Leftrightarrow \mathbf{B}_i y_i = p_i$$
$$v_i = 0 \Leftrightarrow -p_i \leq \mathbf{B}_i y_i \leq p_i$$

Primal-Dual Algorithm:
- Start with an arbitrary choise of a spanning tree ($n - m$ residuals are zero)
- Compute remaining residuals $\mathbf{v}$ (primal variables)
- Obtain flow $\mathbf{y}$ (dual variables) due to the complementary slackness conditions
- If all arcs fulfill capacity bounds an optimal solution is found
- Otherwise eliminate a non-feasible arc and choose a new start solution

Simulation

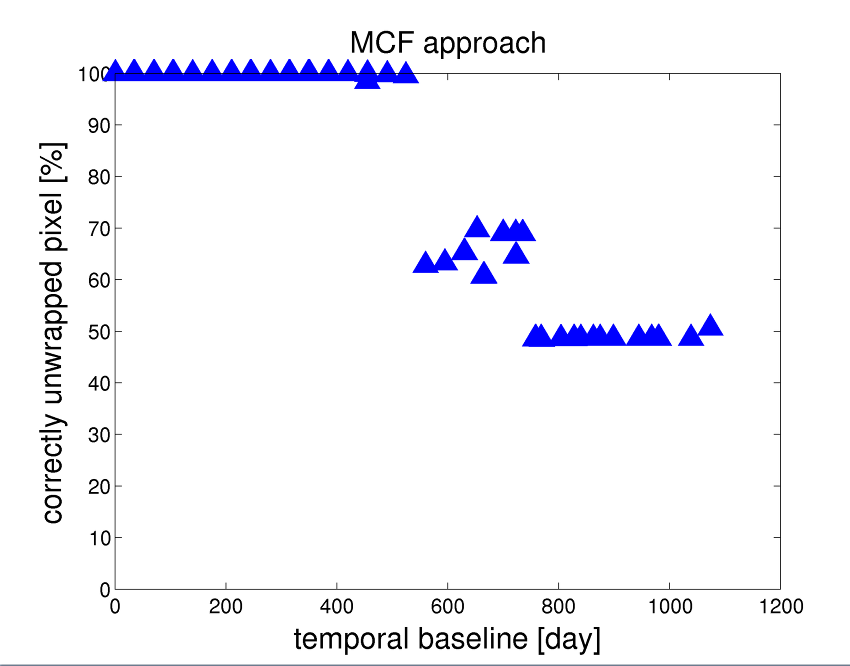
- 191 interferograms are simulated with RSG (Remote Sensing Package Graz)
- Based on ERS 1/2 data (same configuration in time and space) with simulated motion

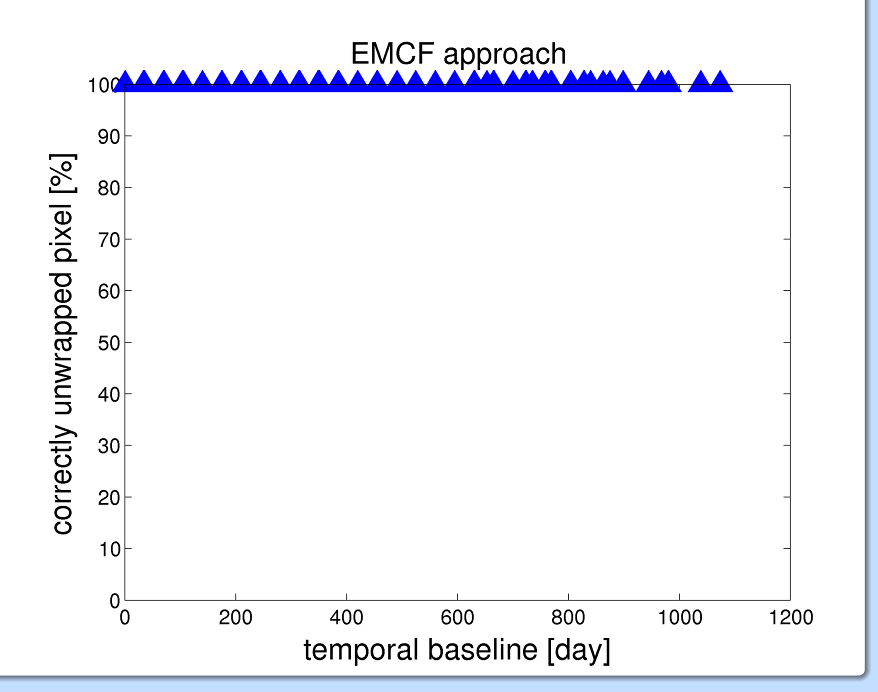




First Results Based on Simulated Data

- Phase Unwrapping gets more error-prone with larger baseline
- Exploiting both the temporal as well as the spatial information results in a higher percentage of correctly unwrapped pixel





Summary, Conclusions and Outlook

- MCF & EMCF approach implemented in Matlab and applied to simulated data
- EMCF approach leads to an improvement
- PhU problem is equivalent to a L1-Norm constrained leveling network
- Future work: application on real data and choise of weights

Acknowledgments

This work was financially supported by the HPSC TerrSys an initiative of the Geoverbund ABC/J. The authors acknowledge the European Space Agency (ESA-Project ID 17055) for the provision of the ERS 1/2 data.

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