Global Approach to Solve the L1-Norm Phase Unwrapping Problem in Differential Radar Interferometry (D-InSAR) Analysis

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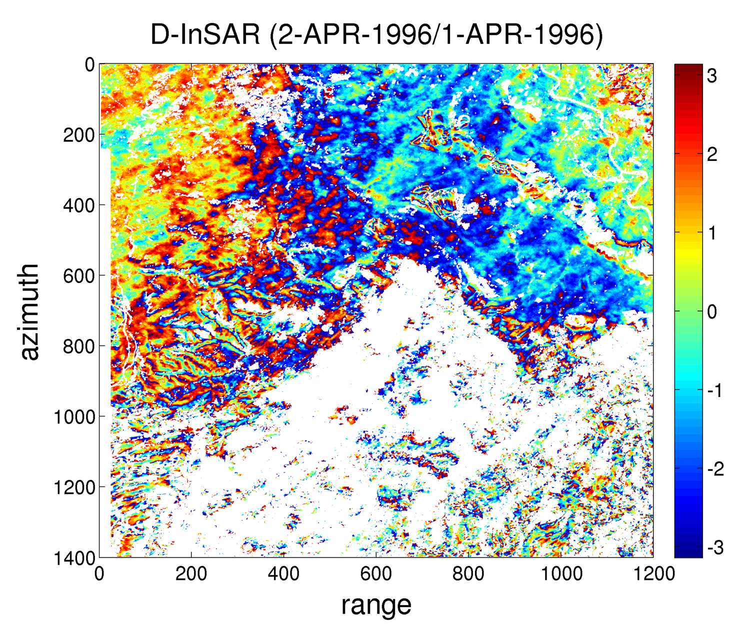
Abstract

The Earth surface is subject to continuously occurring geophysical phenomena. To detect these deformations as well as their temporal behavior differential radar interferometry (D-InSAR) data from several years are used. These data are stacked together and analysed with the Small BAseline Subset (SBAS) method. As with all interferometric applications the problem of phase ambiguities occurs, hence the phase can only be measured modulo 2π. A rather popular technique to reconstruct the phase is the Minimum Cost Flow (MCF) approach. The problem is recast into a network with nodes and arcs searching for the minimum cost flow, defined by the phase ambiguity factors. This can be carried out by choosing the weighted L1-norm for the error criterion. In order to simultaneously unwrap multitemporal D-InSAR data, an extended version to the three dimensional case exists. However, it only works in a step-wise way. The aim of our project is to realize a consistent solution in an one-step algorithm. Therefore, the spatial as well as the temporal information are considered together in one global approach. In this contribution first methodological considerations are shown and applied to simulated data.

Differential Radar-Interferometry (D-InSAR) Analysis



Lower-Rhine-Embayment:

- Rural and brown coal area
  - Open cast-mines: Garzweiler, Hambach and Inden
  - Groundwater lowering

$\rightarrow$ Subaerial surface deformation

Aim:

- Analysis of geophysical phenomena (e.g. deformation time series)

Data:

- Spaceborne radar ERS 1/2 (European Remote Sensing) data from 1992-2000
- Measurement is the interferometric phase $\psi_x^{\Delta t}$

Problem:

- Phase can only be measured modulu 2π:

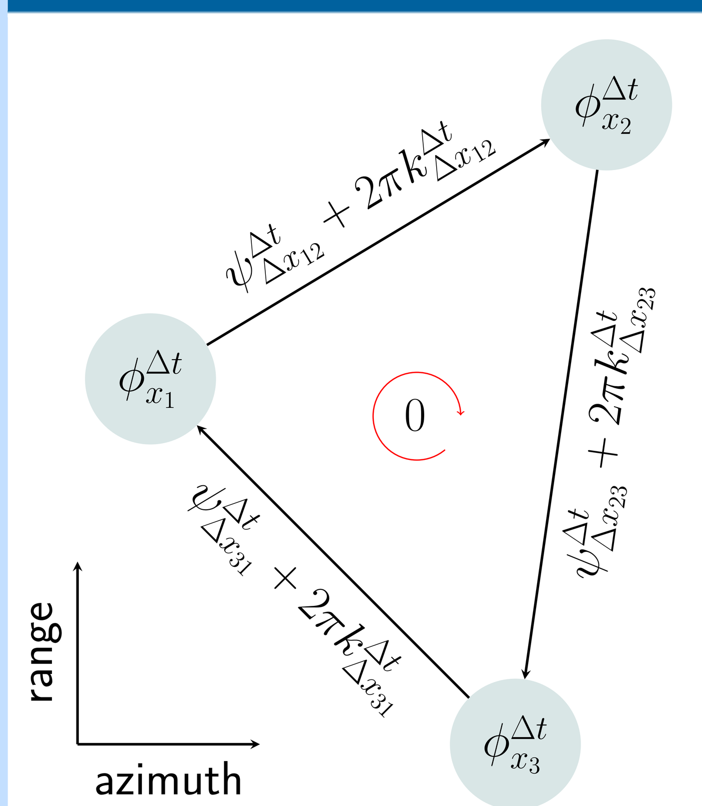
$$\phi_x^{\Delta t} = \psi_x^{\Delta t} + 2\pi k_x^{\Delta t}$$

Phase Unwrapping (PhU) technique:

- Reconstruction with phase gradient
- Recast the problem into a network with nodes and arcs

$\rightarrow$ Searching for the Minimum Cost Flow

Basic Minimum Cost Flow (MCF) Approach



- Basis: single interferogram
- Network: Delaunay triangulation in the azimuth-range plane based on stable scatterer
- Unknown parameter: phase ambiguity factor $k_{\Delta x_{ij}}^{\Delta t}$ along each arc
- Problem formulation:

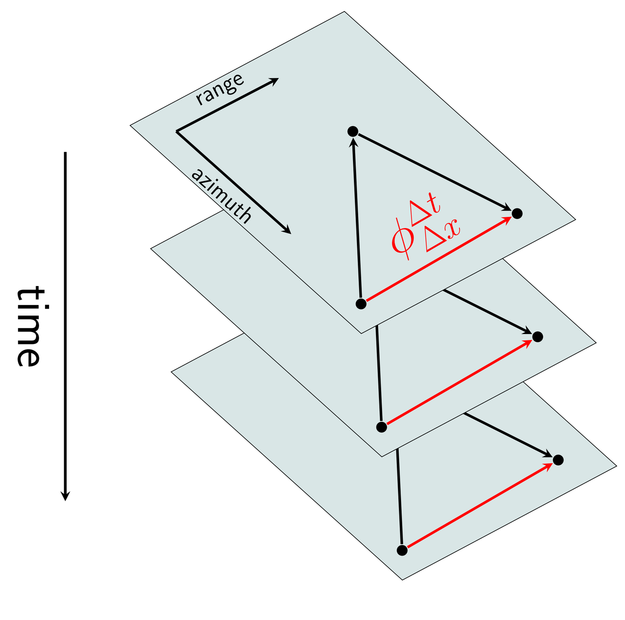
Integer Linear Program in Space

obj. func.: $\boldsymbol{p}^T|\boldsymbol{k}_{\Delta x}^{\Delta t}| \ldots \min$

constraints: $\mathrm{B}_{\text{space}}^T\, \mathbf{k}_{\Delta x}^{\Delta t} = -\lfloor \frac{\mathrm{B}_{\text{space}}^T \psi_{\Delta x}^{\Delta t}}{2\pi} \rceil$

variable: $\mathbf{k}_{\Delta x}^{\Delta t} \in \mathbb{Z}$

Extended Minimum Cost Flow (EMCF) Approach



- Basis: multitemporal D-InSAR images
- Idea: exploitation of the temporal information to bootstrap the basic MCF approach
- Two independent networks:
  1. Spatial Delaunay triangulation
  2. Temporal Delaunay triangulation
- But:
  - Time and space are considered independently
  - Spatial phase unwrapping destroys temporal constraints

1. Temporal Phase Unwrapping

Integer Linear Program in Time

obj. func.: $\boldsymbol{p}^T|\dot{\boldsymbol{k}}_{\Delta x}^{\Delta t}| \ldots \min$

constraints: $\mathrm{B}_{\text{time}}^T\, \dot{\boldsymbol{k}}_{\Delta x}^{\Delta t} = -\lfloor \frac{\mathrm{B}_{\text{time}}^T \boldsymbol{\chi}_{\Delta x}^{\Delta t}}{2\pi} \rceil$

variable: $\dot{\boldsymbol{k}}_{\Delta x}^{\Delta t} \in \mathbb{Z}$

with modified observations

$$\boldsymbol{\chi}_{\Delta x}^{\Delta t} = \boldsymbol{m} + \langle \boldsymbol{\psi}_{\Delta x}^{\Delta t} - \boldsymbol{m} \rangle_{-\pi,\pi}$$

and a specific motion model per arc

$$\boldsymbol{m} = -\tfrac{4\pi}{\lambda}(\boldsymbol{\Delta t} \cdot v_{\Delta x} + \tfrac{\boldsymbol{\Delta b}}{r \sin\vartheta} \cdot \Delta h_{\Delta x})$$

2. Spatial Phase Unwrapping

Integer Linear Program in Space

obj. func.: $\boldsymbol{p}^T|\boldsymbol{k}_{\Delta x}^{\Delta t}| \ldots \min$

constraints: $\mathrm{B}_{\text{space}}^T\, \boldsymbol{k}_{\Delta x}^{\Delta t} = -\lfloor \frac{\mathrm{B}_{\text{space}}^T \dot{\boldsymbol{\phi}}_{\Delta x}^{\Delta t}}{2\pi} \rceil$

variable: $\boldsymbol{k}_{\Delta x}^{\Delta t} \in \mathbb{Z}$

with temporal unwrapped phase gradients

$$\dot{\boldsymbol{\phi}}_{\Delta x}^{\Delta t} = \boldsymbol{\chi}_{\Delta x}^{\Delta t} + 2\pi \dot{\boldsymbol{k}}_{\Delta x}^{\Delta t}$$

Global 3D Phase Unwrapping Approach

- Basis: multitemporal D-InSAR images
- Idea: one global Integer Linear Program with spatial and temporal constraints
- But: motion model is not included and has to be considered in a preprocessing step (from EMCF solution)
- Problem formulation:

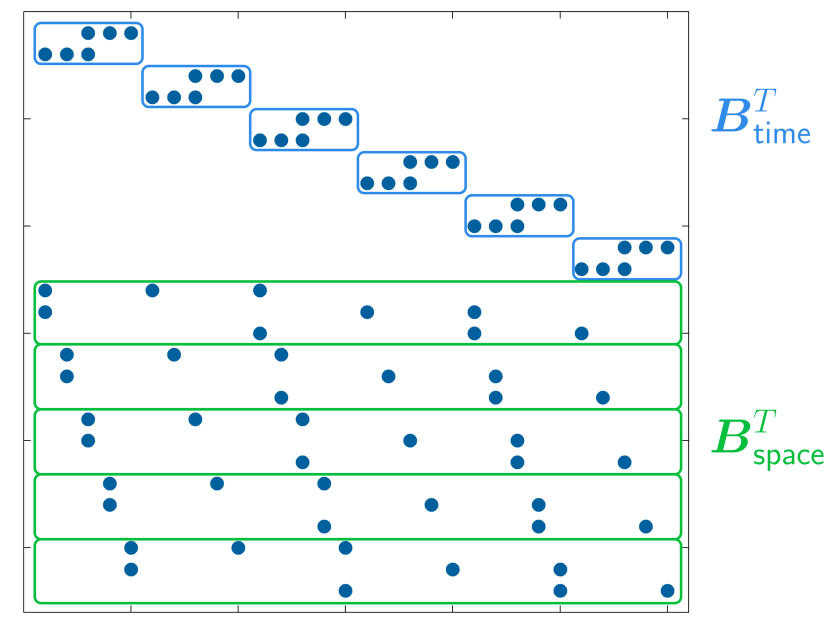
Integer Linear Program in Space and Time

obj. func.: $\boldsymbol{p}^T|\boldsymbol{k}_{\Delta x}^{\Delta t}| \ldots \min$

constraints: $\mathrm{B}_{\text{3D}}^T\, \boldsymbol{k}_{\Delta x}^{\Delta t} = -\lfloor \frac{\boldsymbol{B}_{\text{3D}}^T \boldsymbol{\psi}_{\Delta x}^{\Delta t}}{2\pi} \rceil$

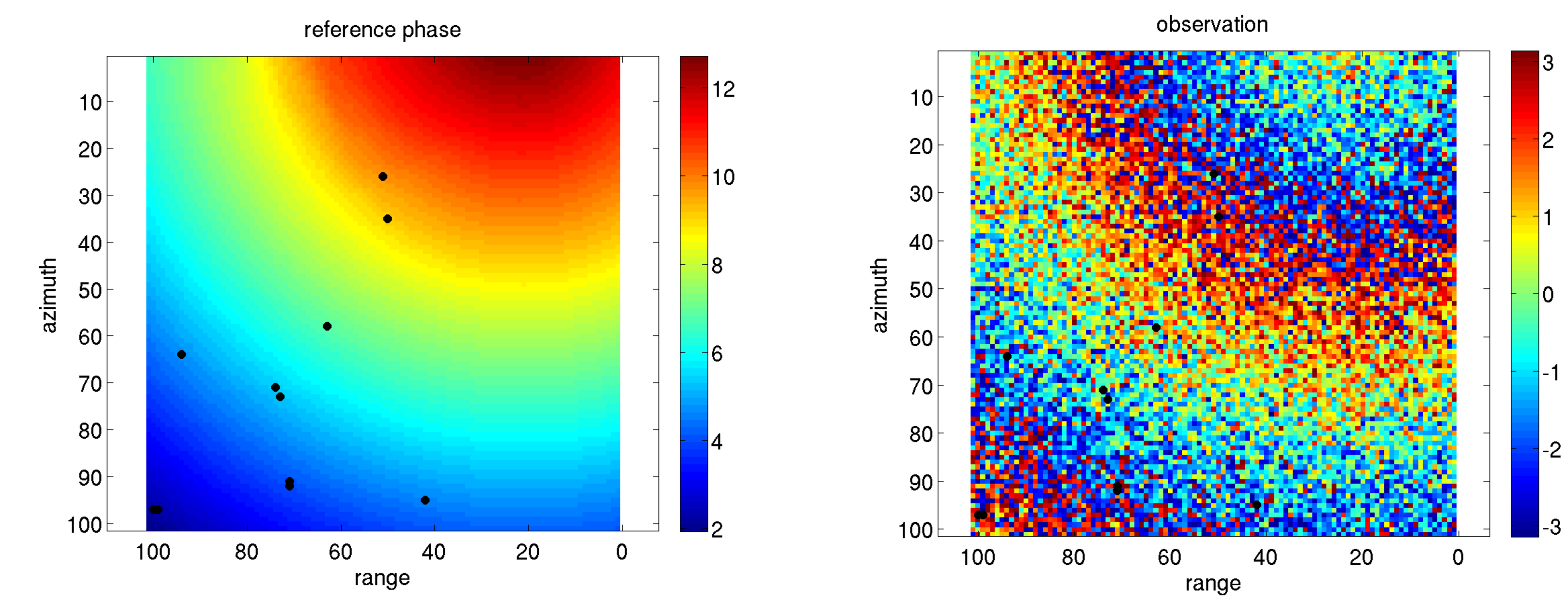
variable: $\boldsymbol{k}_{\Delta x}^{\Delta t} \in \mathbb{Z}$

- Example of the 3D conditional matrix $\boldsymbol{B}_{\text{3D}}^T$:



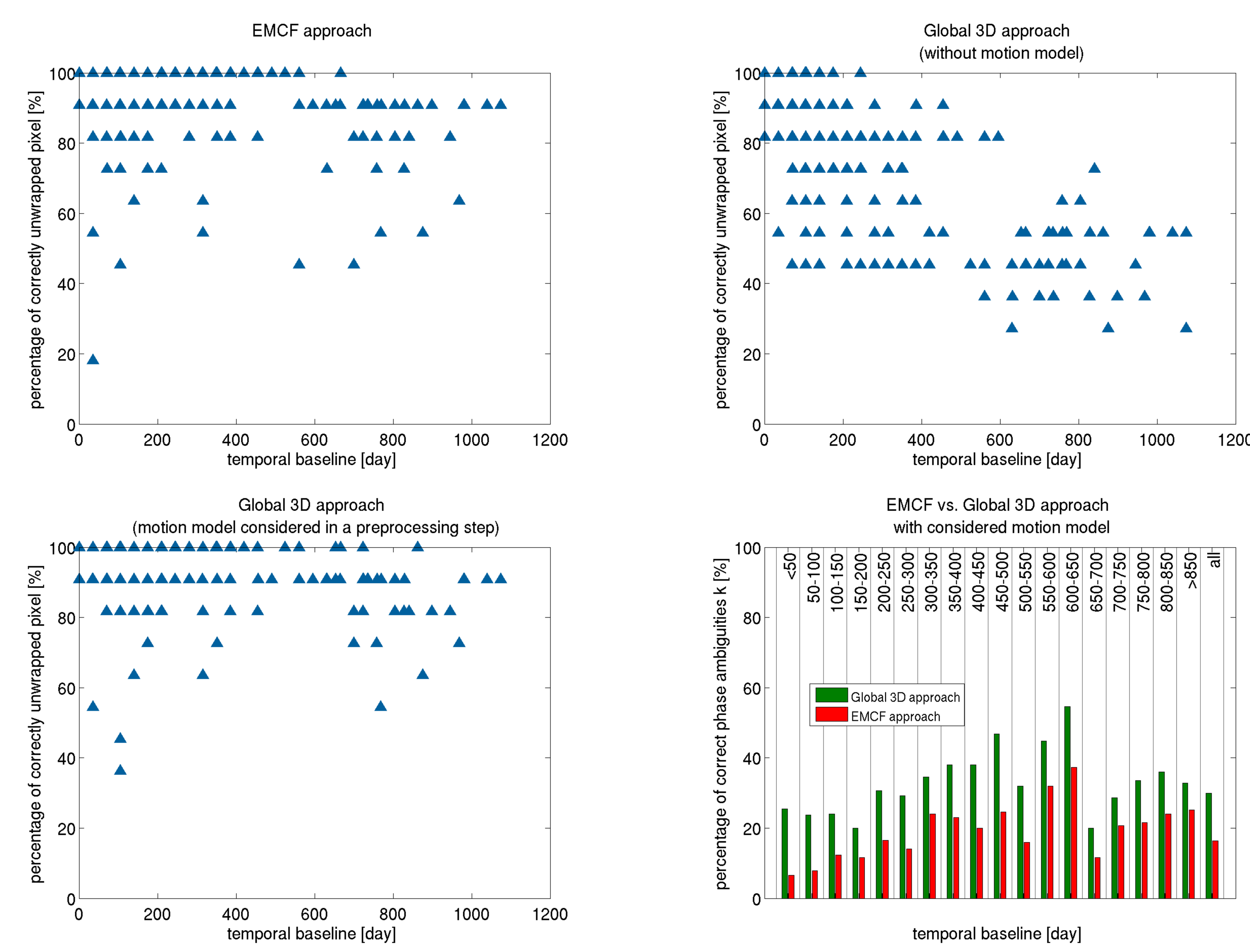
Simulation

- 191 interferograms are simulated with RSG (Remote Sensing Package Graz) and added with standard normal distributed noise
- Based on ERS 1/2 data (same configuration in time and space) with simulated motion



First Results

- EMCF approach provides results which are not consistent in time
- Global 3D Phase Unwrapping approach provides results which are consistent in time and space $\rightarrow$ leads to a higher percentage of correctly unwrapped pixel
- Motion model has to be considered in phase unwrapping approaches



Outlook: Mixed-Integer Approach

- Basis: multitemporal D-InSAR images
- Idea: estimate motion model $\boldsymbol{x} = [\boldsymbol{v}_{\Delta x}\ \Delta\boldsymbol{h}_{\Delta x}]^T$ and phase ambiguity factors $\boldsymbol{k}_{\Delta x}^{\Delta t}$ in one global approach with temporal and spatial constraints
- Problem formulation:
  - Functional model

$$\underbrace{\begin{bmatrix} \psi_{\Delta x_{12}}^{\Delta t_1} \\ \vdots \\ \psi_{\Delta x_{12}}^{\Delta t_N} \\ \psi_{\Delta x_{23}}^{\Delta t_1} \\ \vdots \end{bmatrix}}_{\boldsymbol{\ell}} = \underbrace{\begin{bmatrix} -\frac{4\pi}{\lambda}\Delta t_1 & -\frac{4\pi}{\lambda}\frac{\Delta b_1}{r\sin\vartheta} & 0 & 0 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \ddots \\ -\frac{4\pi}{\lambda}\Delta t_N & -\frac{4\pi}{\lambda}\frac{\Delta b_N}{r\sin\vartheta} & 0 & 0 & \ldots \\ 0 & 0 & -\frac{4\pi}{\lambda}\Delta t_1 & -\frac{4\pi}{\lambda}\frac{\Delta b_1}{r\sin\vartheta} & \ldots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}}_{\boldsymbol{A_1}} \underbrace{\begin{bmatrix} v_{\Delta x_{12}} \\ \Delta h_{\Delta x_{12}} \\ v_{\Delta x_{23}} \\ \Delta h_{\Delta x_{23}} \\ \vdots \end{bmatrix}}_{\boldsymbol{x}} + \underbrace{\begin{bmatrix} -2\pi & \ldots & 0 & 0 & \ldots \\ \vdots & \ddots & \vdots & \vdots & \ddots \\ 0 & \ldots & -2\pi & 0 & \ldots \\ 0 & \ldots & 0 & -2\pi & \ldots \\ \vdots & \ddots & \vdots & \vdots & \ddots \end{bmatrix}}_{\boldsymbol{A_2}} \underbrace{\begin{bmatrix} k_{\Delta x_{12}}^{\Delta t_1} \\ \vdots \\ k_{\Delta x_{12}}^{\Delta t_N} \\ k_{\Delta x_{23}}^{\Delta t_1} \\ \vdots \end{bmatrix}}_{\boldsymbol{k}_{\Delta x}^{\Delta t}} + \underbrace{\begin{bmatrix} e_{\Delta x_{12}}^{\Delta t_1} \\ \vdots \\ e_{\Delta x_{12}}^{\Delta t_N} \\ e_{\Delta x_{23}}^{\Delta t_1} \\ \vdots \end{bmatrix}}_{\boldsymbol{e}_{\Delta x}^{\Delta t}}$$

  - Constraints

$$\mathrm{B}_{\text{3D}}^T\, \boldsymbol{k}_{\Delta x}^{\Delta t} = -\lfloor \frac{\boldsymbol{B}_{\text{3D}}^T\, \boldsymbol{\ell}}{2\pi} \rceil$$

  - Objective funtion

$$\boldsymbol{p}^T|\boldsymbol{k}_{\Delta x}^{\Delta t}| + \boldsymbol{p}^T|\boldsymbol{e}_{\Delta x}^{\Delta t}| \ldots \min$$

  - Variables

$$\boldsymbol{x}, \boldsymbol{e} \in \mathbb{R}, \boldsymbol{k}_{\Delta x}^{\Delta t} \in \mathbb{Z}$$

- Task: reformulation as Linear Program with mixed-integer variables via primal/dual relationship

Summary, Conclusions and Outlook

- Time and space are considered independently in step-wise EMCF approach $\rightarrow$ results are not consistent in time
- Global 3D Phase Unwrapping approach leads to an improvement, but motion model has to be considered in a preprocessing step
- Future work: Global 3D Mixed-Integer approach with phase ambiguities and motion model

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