Fringe 2017

Integrated Spatio-temporal Estimation of a Deformation Time-series from a Stack of Unwrapped Differential Interferograms

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June 7, 2017

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Spatio-temporal Signal Separation

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Basic Idea

Input:

unwrapped, differential image stack

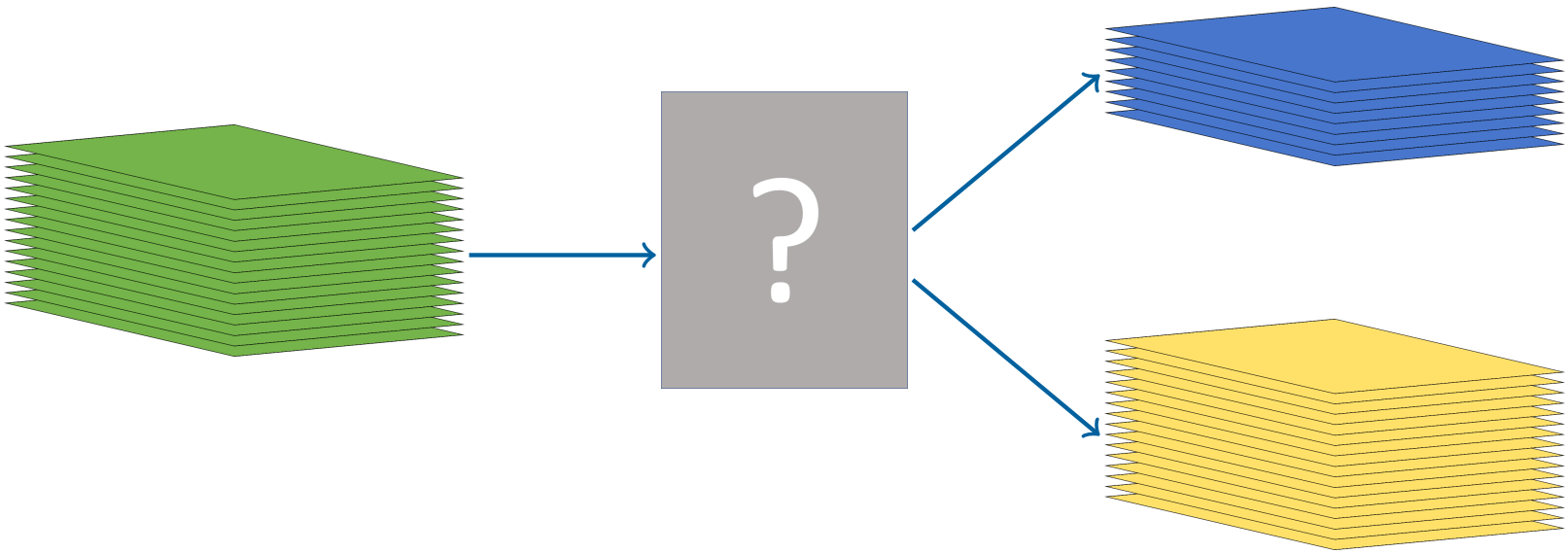
Spatio-temporal filter:

one-step approach

Output:

1) filtered images for each acquisition date (e.g. deformation)

2) differential, residual signal (e.g. atmosphere)



Demands on the Filter Design

1. How to design a filter under following conditions?
   - unregular sampling rate (e.g. due to missing scenes)
   - unregular (clustered) sampling in space
2. How do we link the temporal and spatial filtering step?
3. How can we include the stack inversion?

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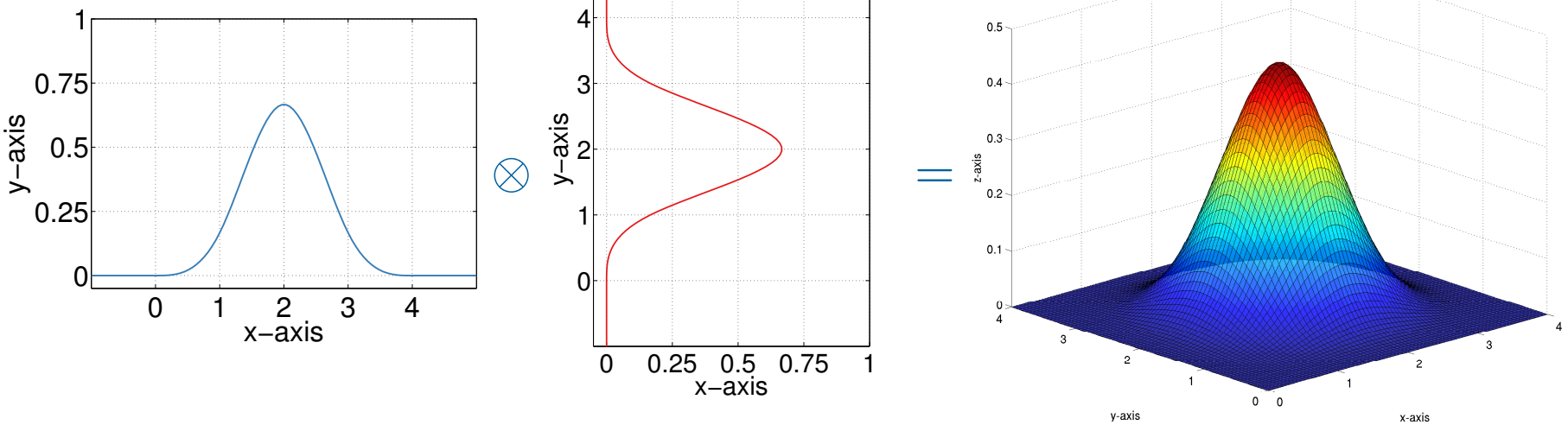
Tensor-B-splines and least squares adjustment!

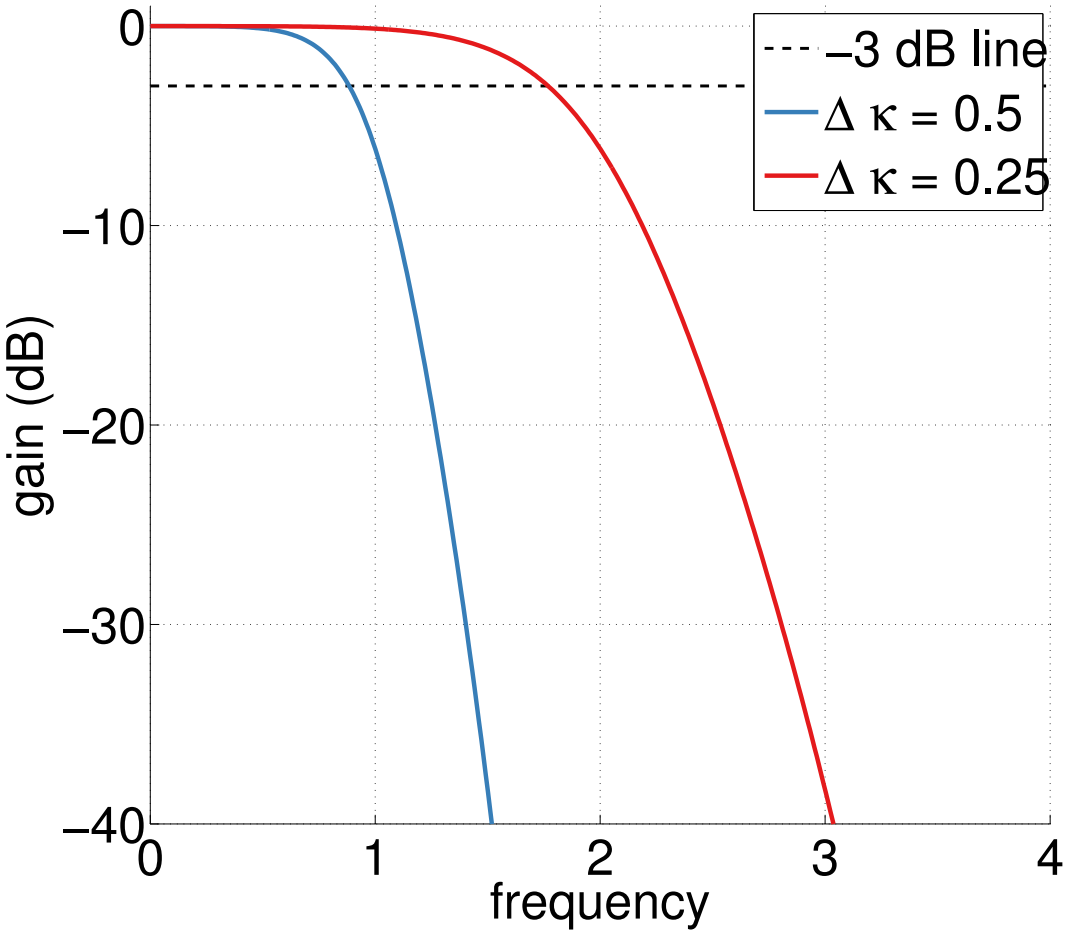


Outline

Spectral Properties of B-Splines

Spectral properties of a linear combination of **equidistant** B-splines are defined by

1. the spacing $\Delta\kappa$



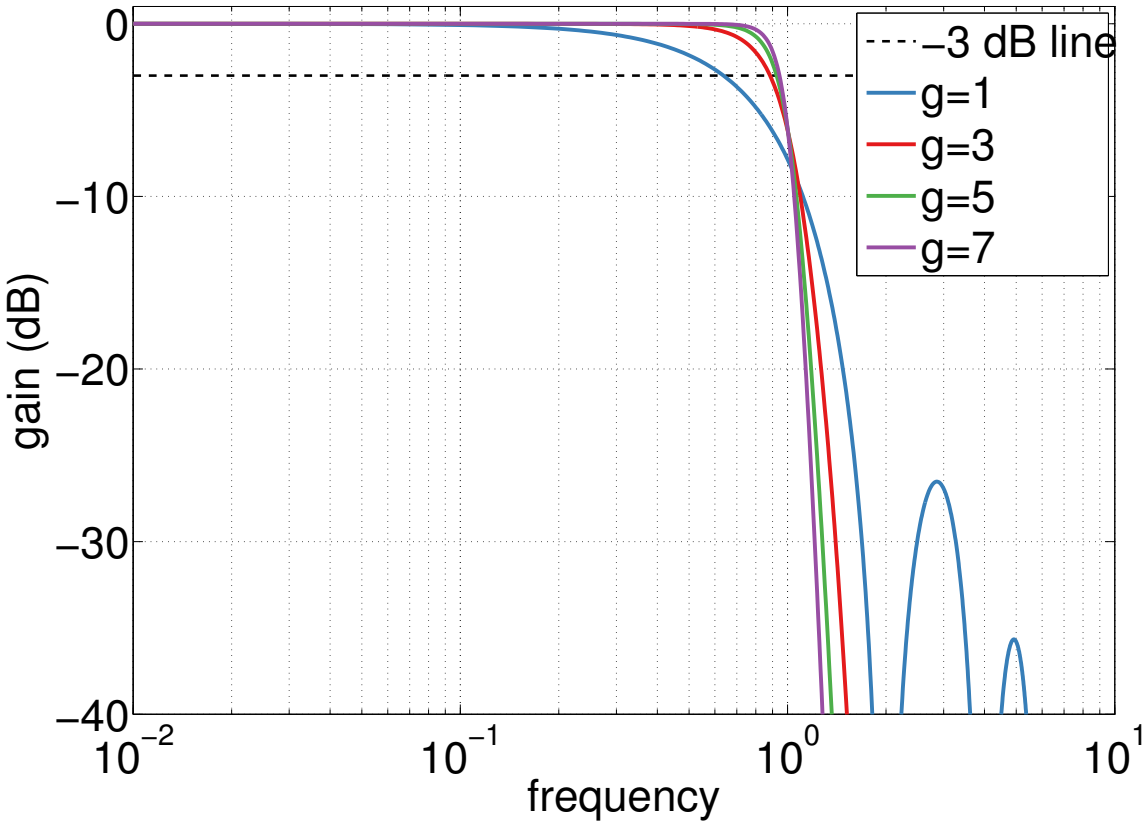
Rule of Thumb:

$$f_{\text{cutoff}} \approx \frac{1}{2\Delta\kappa}$$

Spectral Properties of B-Splines

Spectral properties of a linear combination of **equidistant** B-splines are defined by

1. the spacing $\Delta\kappa$ and
2. the order g of the B-spline.

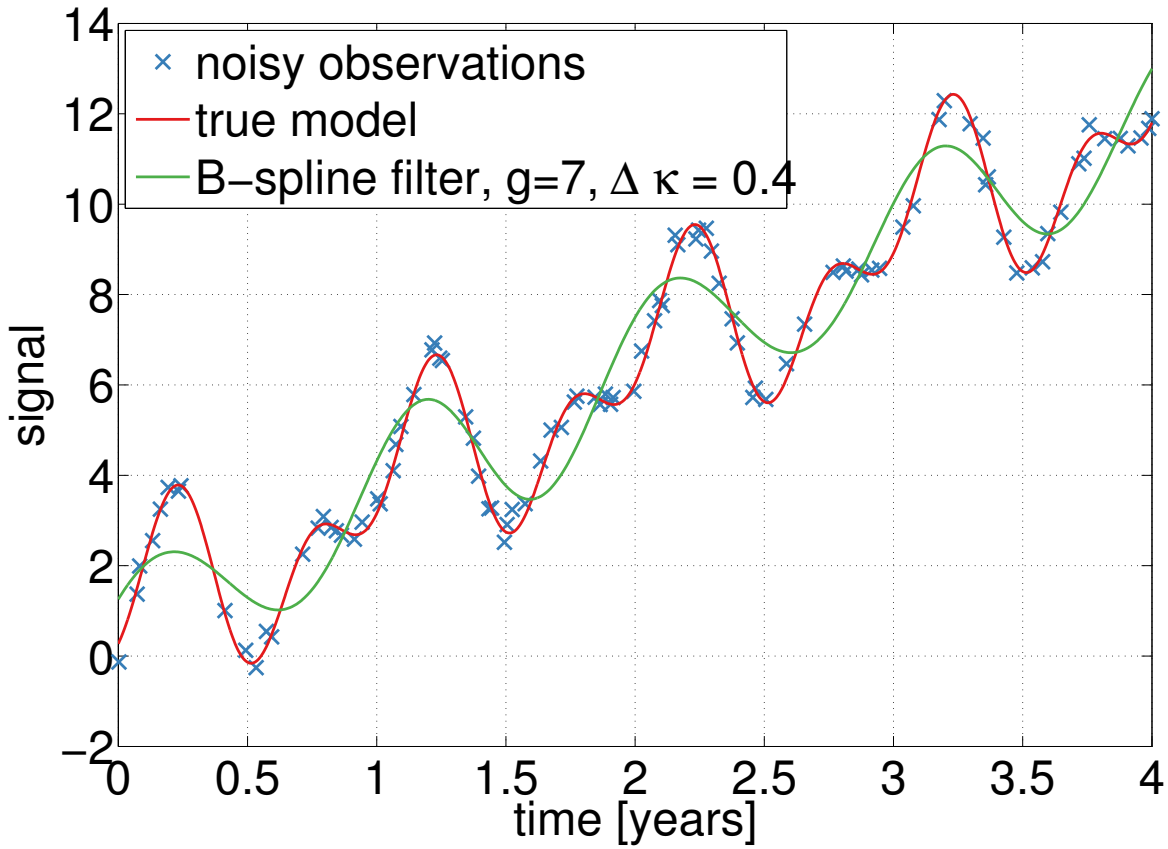


a) higher order $\rightarrow$ narrower transition zone

b) "poor" performance for linear B-splines

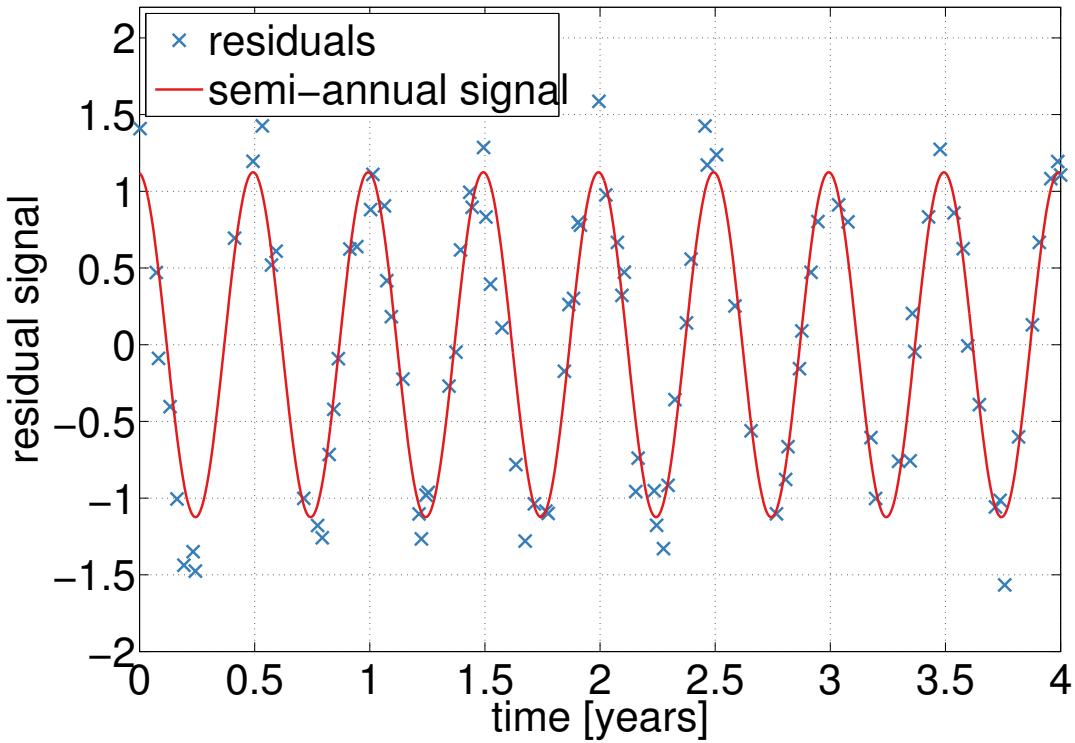
Filtering a Seasonal Signal

1. Simulation of a linear trend superimposed with seasonal variations
2. Non-equidistant and noisy data ($\sigma = 0.2$)
3. Filter out semi-annual variations with B-spline model



Filtering a Seasonal Signal

1. Simulation of a linear trend superimposed with seasonal variations
2. Non-equidistant and noisy data ($\sigma = 0.2$)
3. Filter out semi-annual variations with B-spline model
4. Analyzing the residuals



Spatio-temporal Model

- Representation of deformation signal using bivariate Tensor-B-splines of order g with time-dependent coefficients:

$$f(x, y, t) = \sum_{i=-g}^{p-1} \sum_{j=-g}^{q-1} a_{ij}(t) B_i^g(x) B_j^g(y),$$

p, q ... number of nodes inside the area of interest.

- Time-dependent coefficients as frequency-selective B-spline model:

$$a_{ij}(t) = \sum_{k=-g}^{r-1} a_{ijk} B_k^g(t),$$

r ... number of nodes in the acquisition period.

Observation Equation

Observation equation for least squares adjustment:

$$\boldsymbol{\ell}_i + \boldsymbol{v}_i = (\boldsymbol{S}_i\boldsymbol{T}) \otimes (\boldsymbol{Y}_i \otimes \boldsymbol{X}_i)\,\boldsymbol{a} = \boldsymbol{A}_i\boldsymbol{a}$$

$\boldsymbol{\ell}_i$	...	i-th observation,
$\boldsymbol{v}_i$	...	i-th residual,
$\boldsymbol{T}$	...	design matrix for temporal model,
$\boldsymbol{S}_i$	...	row of the matrix for stack inversion,
$\boldsymbol{X}_i$	...	row of the design matrix of the spatial model along the x-axis,
$\boldsymbol{Y}_i$	...	row of the design matrix of the spatial model along the y-axis,
$\boldsymbol{A}_i$	...	i-th of the design matrix of the combined model,
$\boldsymbol{a}$	...	unknown parameters.

Resolving the Rank Deficiency

Problem: phase differences $\rightarrow$ model is rank deficient!

- We overcome the rank deficiency by the common assumption

$$f(x, y, t_{ref}) = \sum_{i=-g}^{p-1} \sum_{j=-g}^{q-1} a_{ij}(t_{ref}) B_i^g(x) B_j^g(y) \stackrel{!}{=} 0, \qquad \forall x, y \in \mathbb{R}^2$$

$\rightarrow$ i.e. signal (e.g. deformation) vanishes at time t_{ref}.

Final step: filtering with a restricted least squares approach

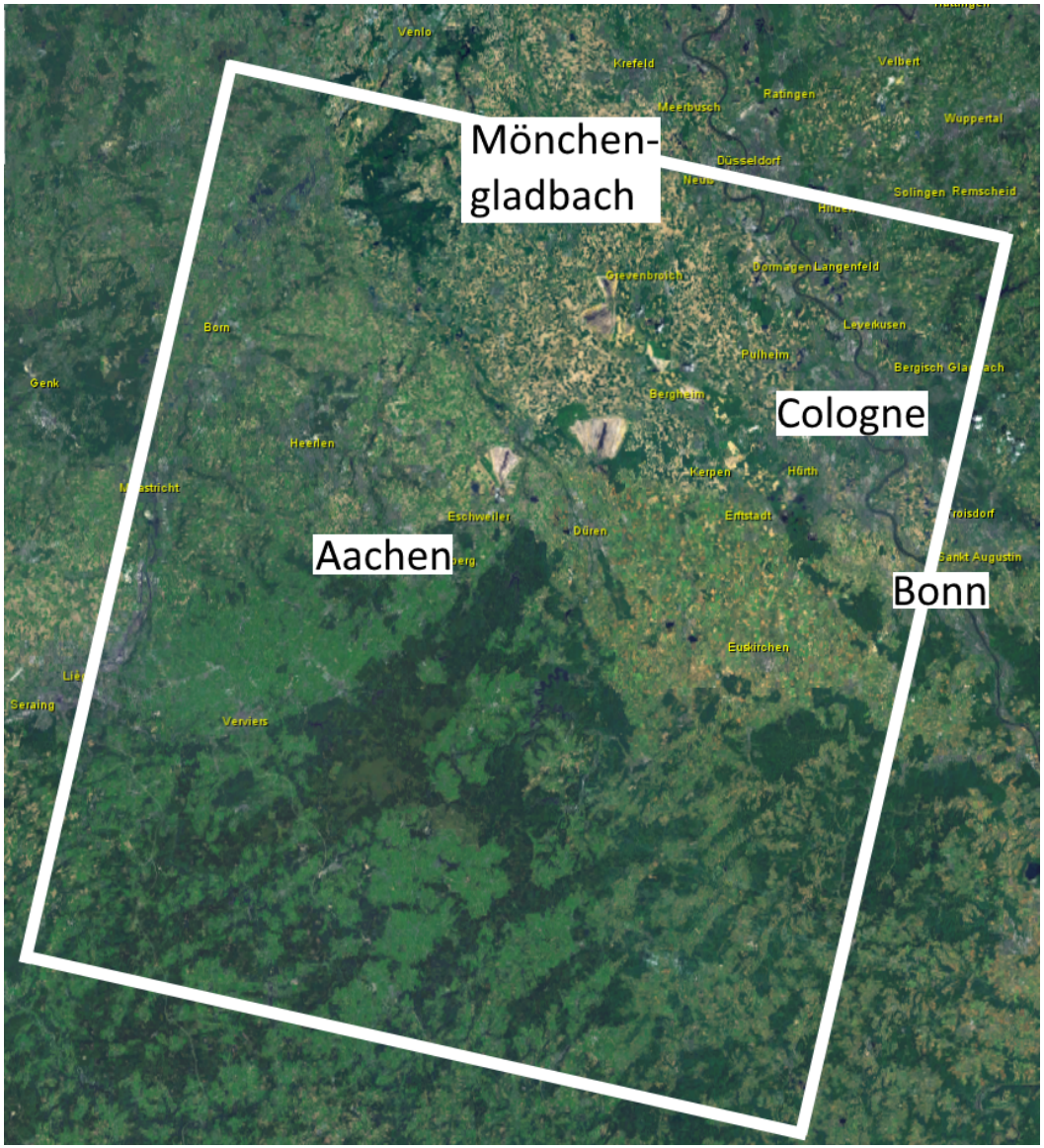
$$\begin{bmatrix} \boldsymbol{A}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{B}^T & \boldsymbol{\Phi} \end{bmatrix} \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{k} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\ell} \\ \boldsymbol{0} \end{bmatrix},$$

$\boldsymbol{B}$ ... resolves the rank deficiency (so that $\boldsymbol{B}^T \boldsymbol{a} = \boldsymbol{0}$),

$\boldsymbol{k}$ ... vector with Lagrange multipliers and

$\boldsymbol{\Sigma}$ ... optional co-/variance information.

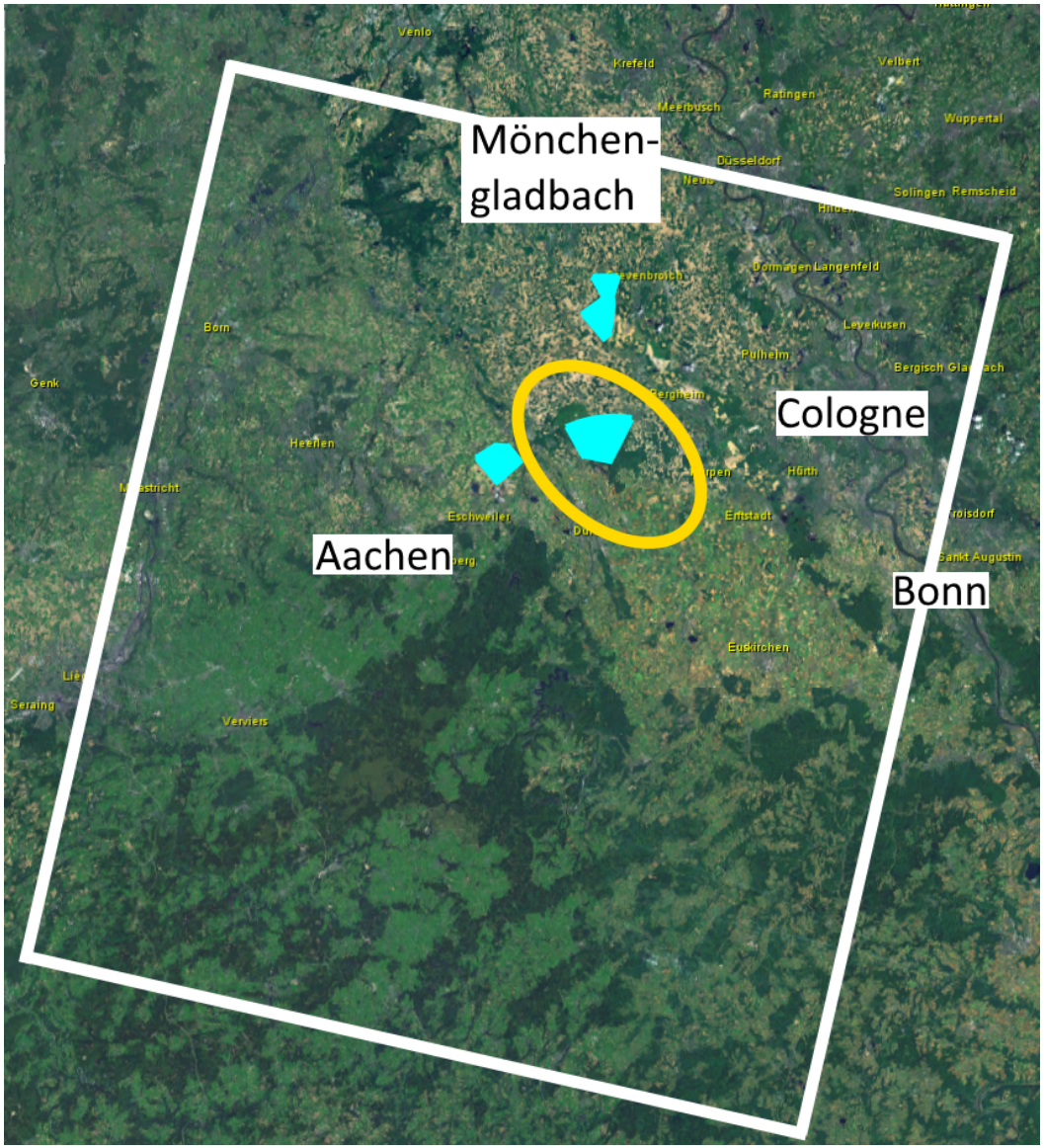
Area of Investigation



Key features:

- coal-mining area near Cologne, Germany

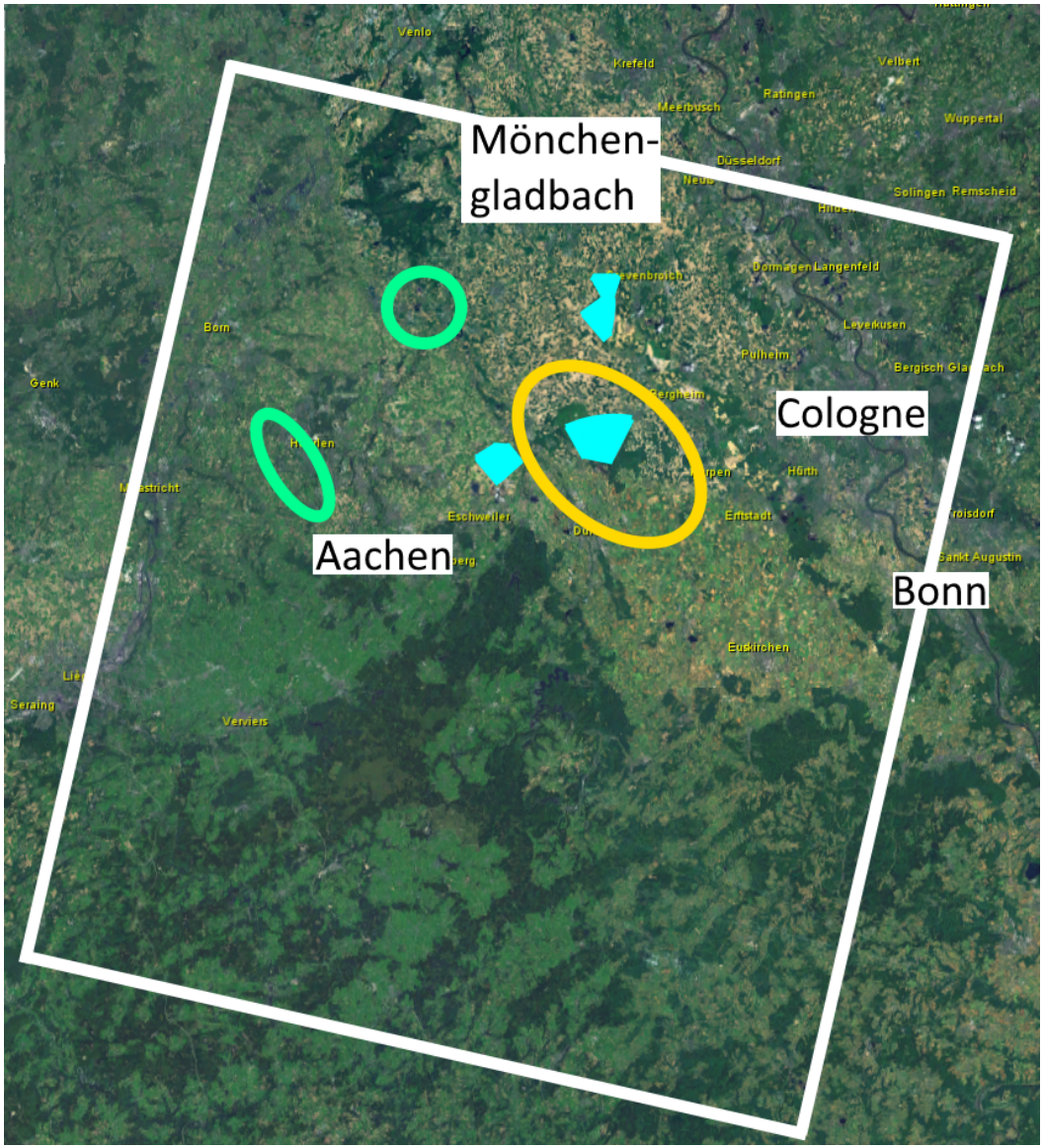
Area of Investigation



Key features:

- coal-mining area near Cologne, Germany
- 3 active open-cast mines
  $\rightarrow$ subsidence

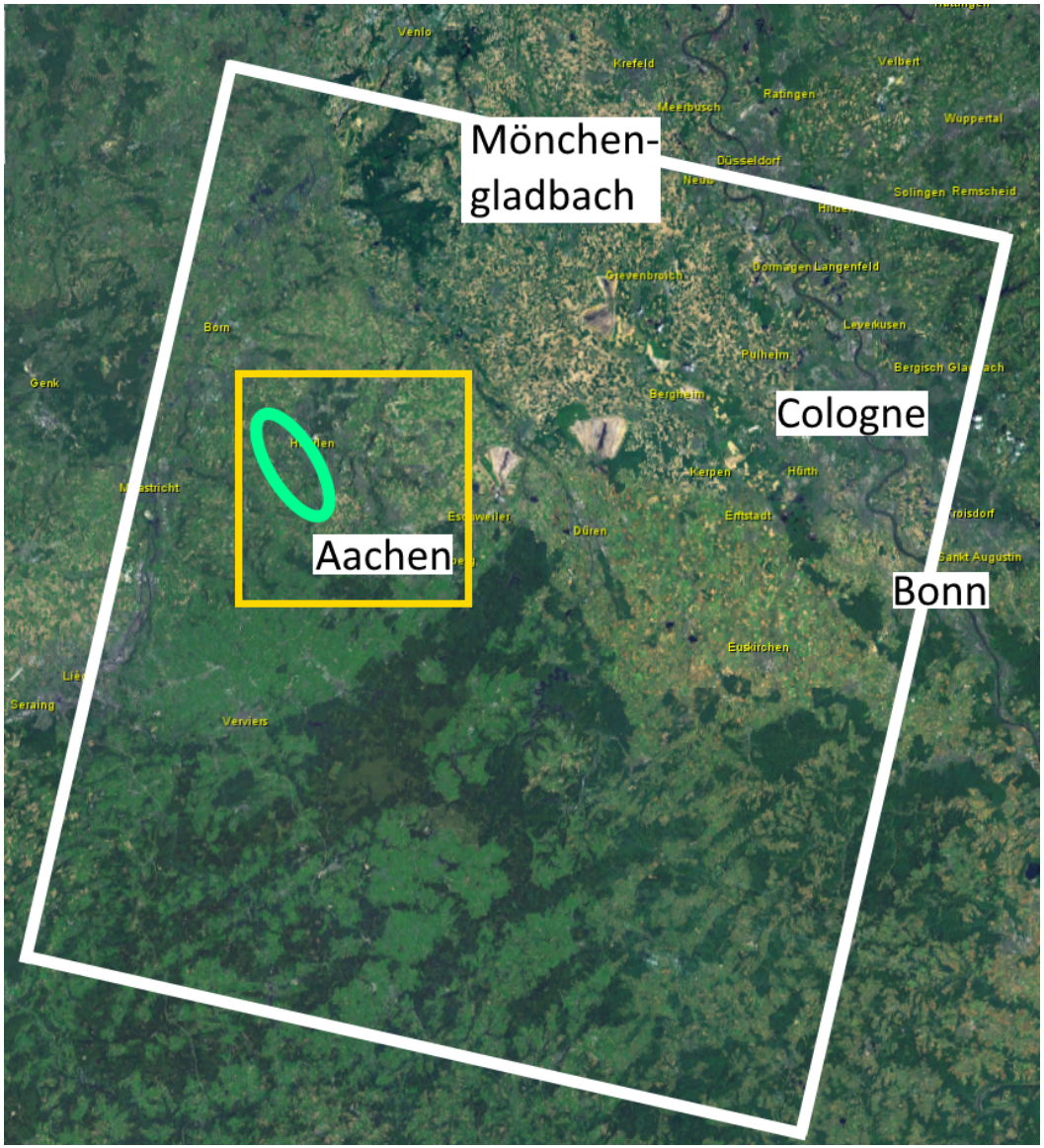
Area of Investigation



Key features:

- coal-mining area near Cologne, Germany
- 3 active open-cast mines
  → subsidence
- 2 inoperative mining areas
  → uplift

Area of Investigation

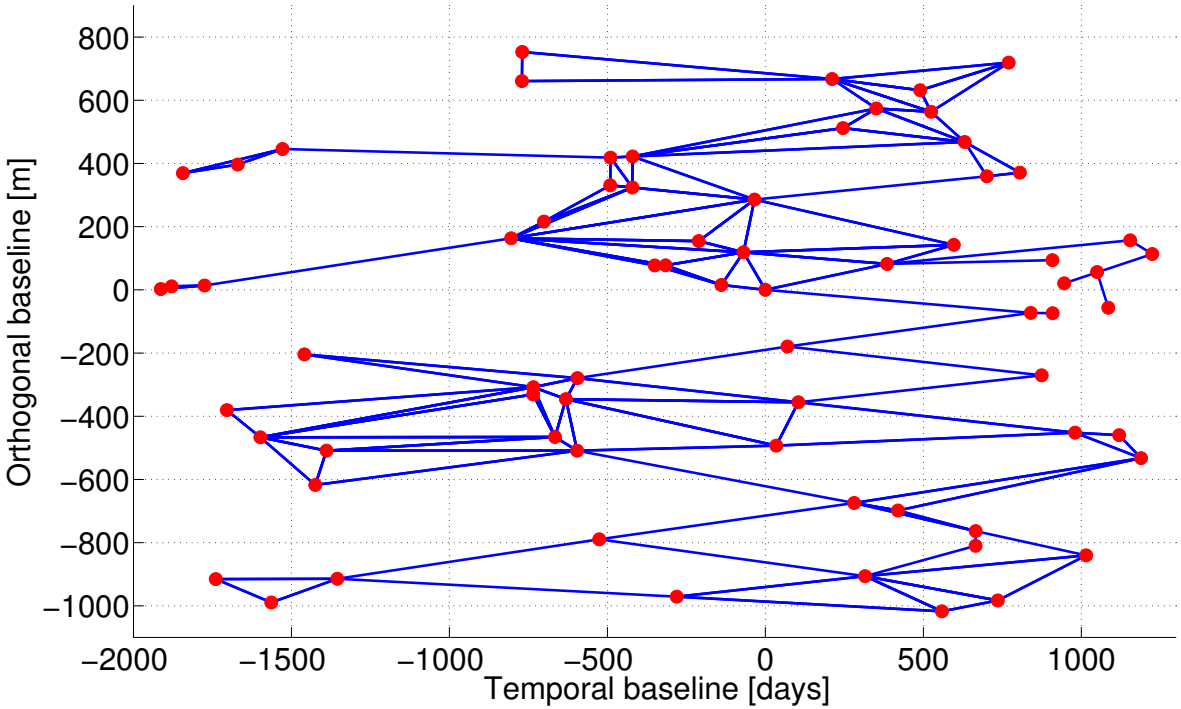


Key features:

- coal-mining area near Cologne, Germany
- 3 active open-cast mines
  → subsidence
- 2 inoperative mining areas
  → uplift
- interferograms strongly decorrelated
  → test area

Data

- 71 ERS-1/2 acquisitions (May 1992 to Dec. 2000)
- 127 SBAS interferograms

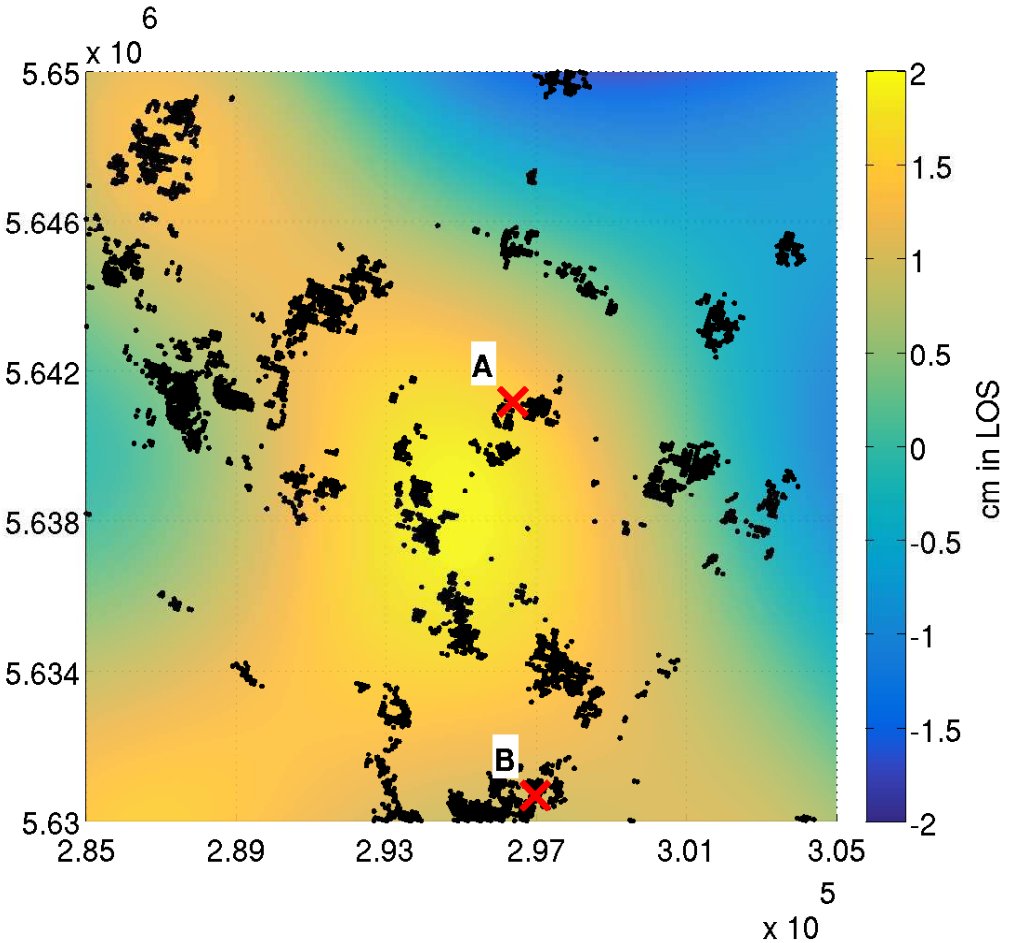


- *Improved EMCF-SBAS Processing Chain* (Pepe et al. 2015)
- Software: *RSG* (Joanneum Research)
- no atmospheric filtering and no stack inversion

Estimated Ground Movement

Ground movement between March 1993 and March 1997:

- A: leveling point in deformation area
- B: leveling point in non-moving area



Setting:

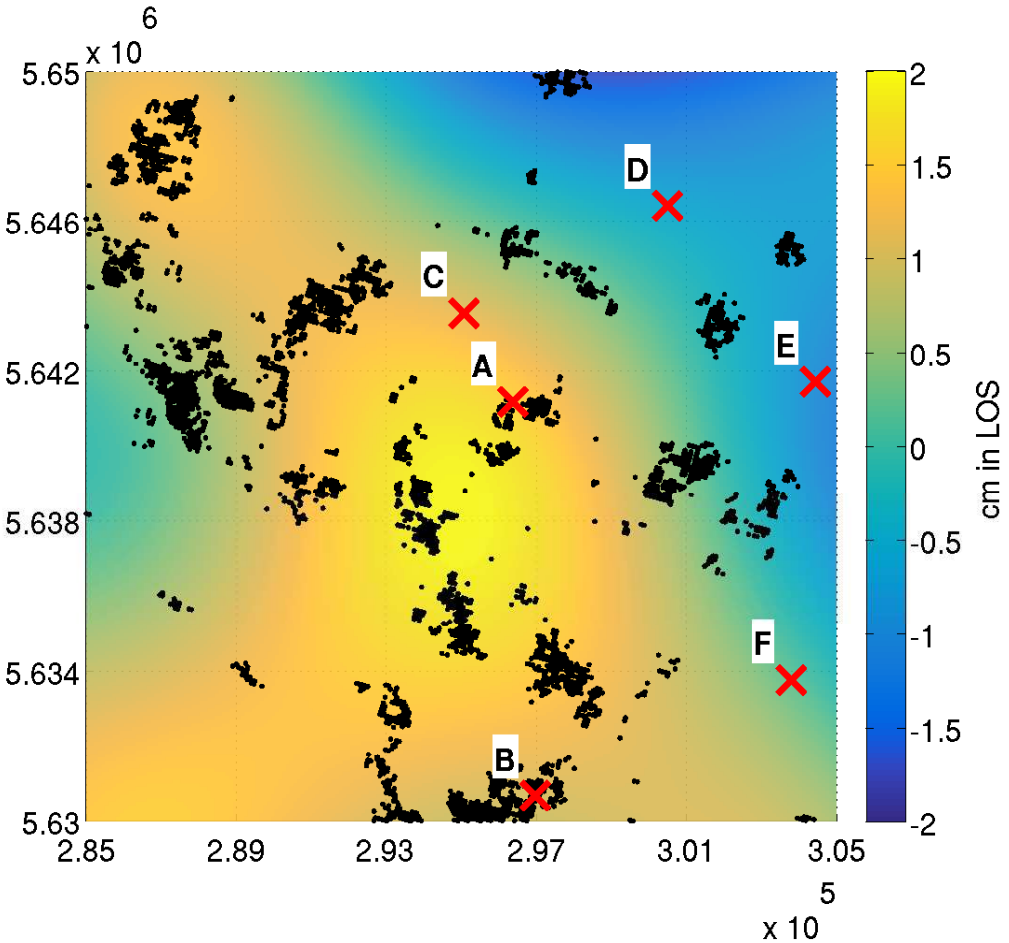
- $\Delta\kappa_t \approx 1.5$ years
- $\Delta\kappa_x = \Delta\kappa_x = 10$ km
- B-spline order $g = 3$ in space and time

Leveling Point	Leveled Δh	Estimated Δh
A	1.5 cm	1.4 cm
B	0.1 cm	0.2 cm

Estimated Ground Movement

Ground movement between March 1993 and March 1997:

- A+B: leveling points near scatterers
- C-F: leveling points in decorrelated areas



Leveling Point	Leveled Δh	Estimated Δh
A	1.5 cm	1.4 cm
B	0.1 cm	0.2 cm
C	1.8 cm	1.1 cm
D	-0.2 cm	-0.4 cm
E	-1.5 cm	-1.2 cm
F	0.7 cm	1.1 cm

Conclusion

Key features of the proposed method:

- signal separation by the use of a spatio-temporal lowpass filter,
- one-step approach,
- estimation of an continuous function in space and time.

Current work:

- data gaps,
- boundary values,
- refinement of stochastic model,
- multi-mission analysis,
- ...