Parametric finite element based models to represent the Mean Dynamic Ocean Topography

Christian Neyers, Jan Martin Brockmann and Wolf-Dieter Schuh

Institute of Geodesy and Geoinformation, Theoretical Geodesy Group, University of Bonn

Abstract

The difference between the sea surface height (SSH) above a reference ellipsoid and the geoid is the ocean's dynamic topography (DOT), which reflects many characteristics of the ocean circulation. Its steady-state part is called the mean dynamic ocean topography (MDT). The geodetic estimation of the DOT (or MDT) is typically based on the separation of altimetric SSH measurements into the MDT and the geoid. For the separation process, additional independent information about the geoid, the MDT or both is required. A remaining scientific issue is the spectral inconsistency of the involved data sets.

In contrast to the well established multi-step procedures, we propose a parametric representation of the MDT and its estimation in an integrated one step approach. In this contribution we will demonstrate that a parametric finite element based model is well suited to represent the signal of the MDT. We concentrate on finite elements which are able to represent a signal either C^0- or C^1-smooth. Additionally, different observation equations for various functionals sensitive to the MDT signal - like along-track satellite altimetry, (radial SAR) surface velocities or velocities derived by surface drifters - can be formulated. We will summarize the status of the DFG funded project PARASURV, where an integrated approach of finite element based MDT estimation is developed (special focus on use of radial SAR derived surface velocities).

Motivation and Introduction

Typical geodetic DOT estimation: Difference of a Mean Sea Surface (MSS) model and a geoid (N) with subsequent filtering (e.g. [1, 6])

$$\zeta = \mathbf{F}\,(MSS - N) \qquad (1)$$

Problems of typical approaches

- working grid defined by MSS grid
- multi-step procedures
- MSS model's lack of an error model
- the used geoid model is often not independent
- the geoid error is not accounted for
- integration of complementary data, e.g. surface currents is difficult
- multi-mission analysis: defined by MSS

Our approach to overcome deficits

- ⇒ continuous parametric model
- ⇒ integrated estimation: DOT + geoid
- ⇒ along-track obs. + uncertainty
- ⇒ use satellite based geoid models, independent of altimetry
- ⇒ use spherical harmonics + full VCM
- ⇒ advanced observation equations due to continuous model
- ⇒ continuous model

Parametric approach for the DOT estimation [3, 4, 5, 8]

geoid/gravity
- observations: satellite-based gravity field NEQ (e.g. GOCO), smoothness
- full VCM + omission
- spherical harmonics

$l_{GF} = f(x_{GF})$

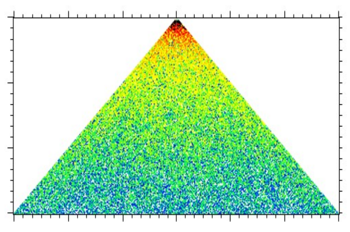
sea surface height
- observations: along-track SSH of multiple missions (e.g. altimetry, **SWOT**)
- empirical error model
- potentially: tide gauges

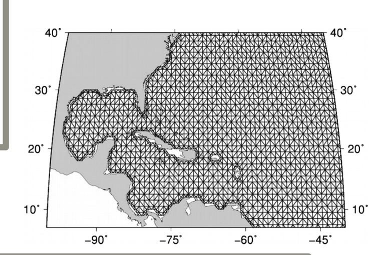
$l_{SSH} = f(x_{GF}, x_{DOT})$

ocean surface
- observations: drifters, surface currents (e.g. **SAR**, **SKIM**, ...), smoothness conditions
- empirical error model

$l_{OG} = f(x_{DOT})$

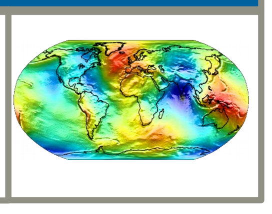
joint least squares adjustment
- estimate gravity field & DOT
- separation of SSH
- weighted combination
- high-dimensional, HPC
- spatial & temporal filtering



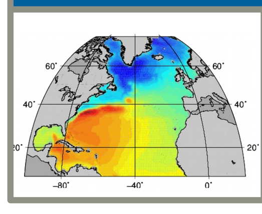


estimated parameters

gravity field
- parametric model: spherical harmonics
- parameters: $x_{GF} = [\,c_{lm}, s_{lm}\,]$
- full VCM



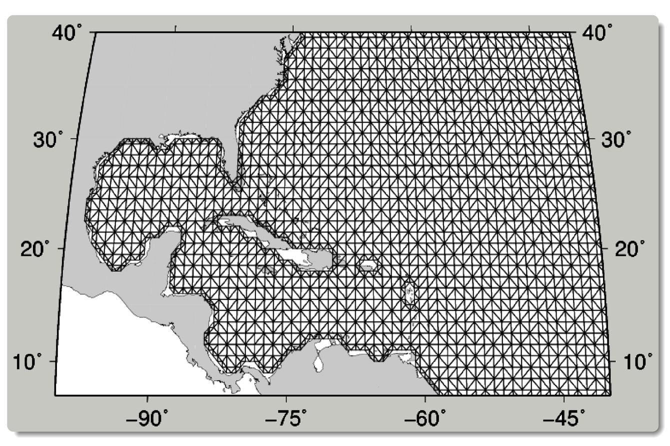
dynamic ocean topography
- parametric model: finite element space
- parameters: $x_{DOT} = [\,a_k(t)\,]$
- full VCM



Finite elements to describe the DOT

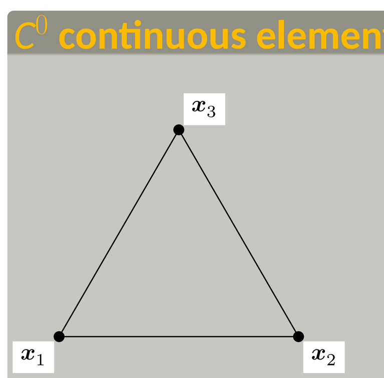
Mean DOT is represented by linear combination of finite element base functions $b_k(\theta, \lambda)$

$$\zeta(\theta, \lambda, t) = \sum_{k \in K} a_k(t)\, b_k(\theta, \lambda), \qquad \mathbf{x}_{DOT} = [a_k(t)] \qquad (2)$$



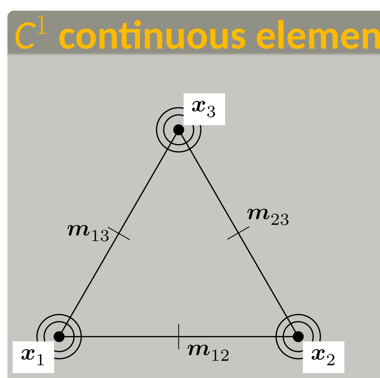
- continuous model in space (C^0/C^1-smooth)
- unknowns a_k interpretable (e.g. DOT, 1st/2nd derivative, ...)
- finite element space defines filtering/spatial resolution
- observation equations in any location and functional (point values, derivatives, integrals)
- optional: modeling time-dependency $a_k(t)$

C^0 continuous element: linear element



- 3 degrees of freedom
- simple: DOT in nodes
- jumps in derivatives
- details: [3]

C^1 continuous element: ARGYRIS element



- 21 degrees of freedom
- C^1 continuous ⇒ integrate currents
- project PARASURV
- details: [2, 9]

Study: Modeling the MDT with finite elements

Proof of concept: represent DOT model with C^0- and C^1-smooth finite elements

Study setup:

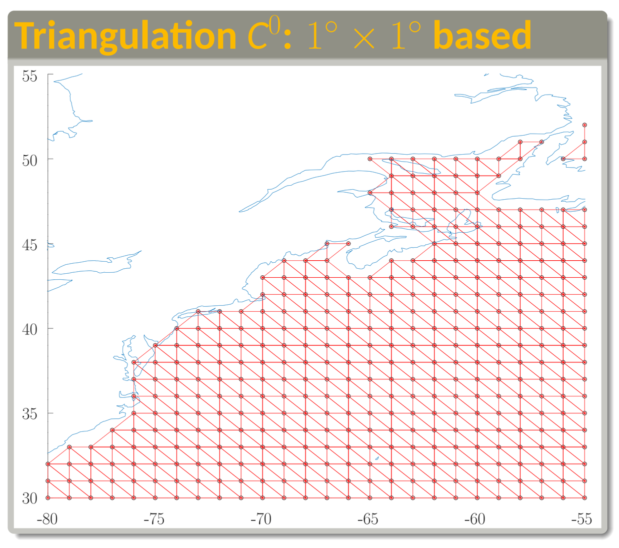
- observations are $0.25°$ gridded points of CNES-CLS13 MDT [10] in North Atlantic (roughly 30 000 points used)
- approximate the signal with different finite element spaces:
  - C^0-smooth: linear element
  - C^1-smooth: Argyris element

  } with comparable spatial resolution
- estimate the a_k for both scenarios in least squares adjustment

⇒ represent signal with finite element space

⇒ implement the Argyris element in a test environment

Triangulation C^0: $1° \times 1°$ based

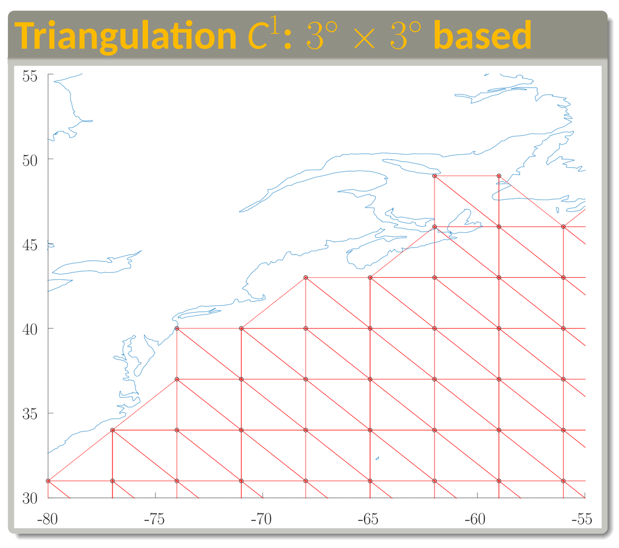


- triangulation of regular grid

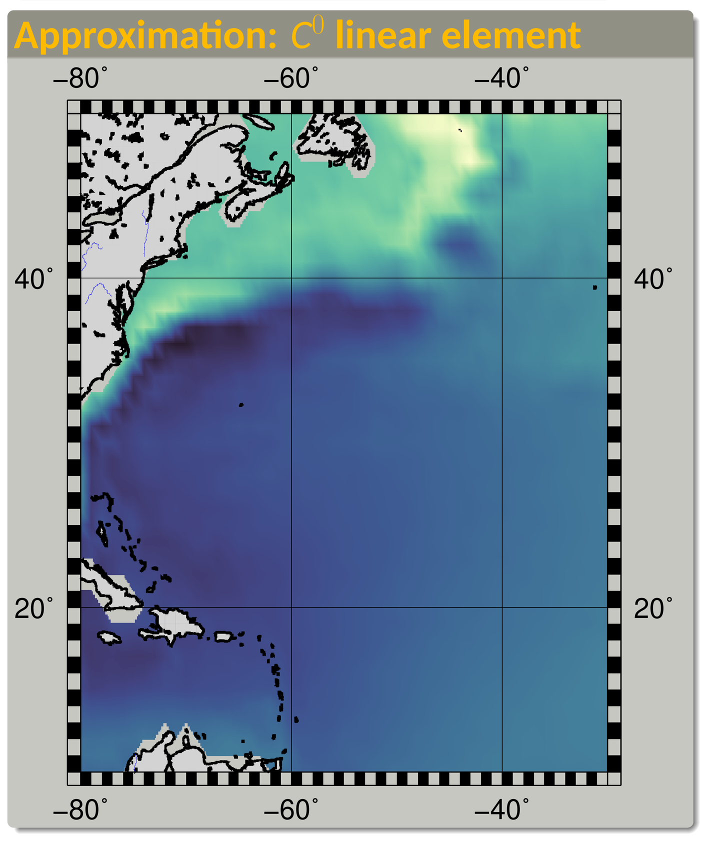
⇒ not optimized

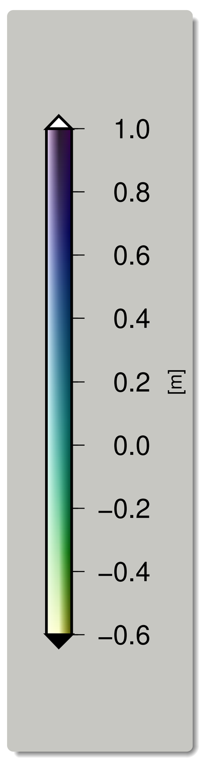
- # parameter C^0: 2 022
- # parameter C^1: 2 045

Triangulation C^1: $3° \times 3°$ based

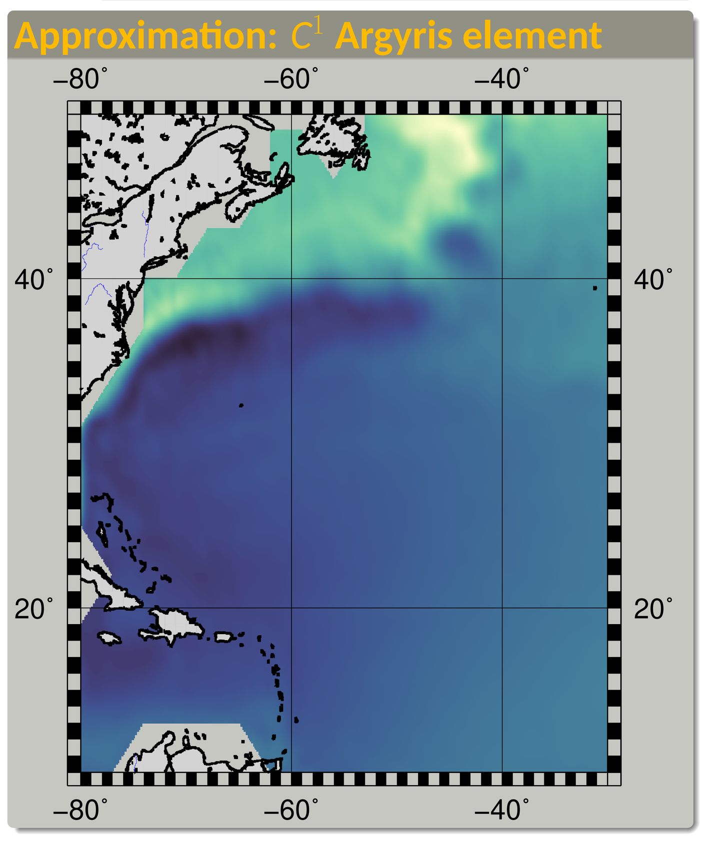


Approximation: C^0 linear element





Approximation: C^1 Argyris element



residual statistics (C^0): min -22.5 cm, max 19.7 cm, $\tilde{\sigma}_o = 1.3$ cm

residual statistics (C^1): min -10.2 cm, max 7.4 cm, $\tilde{\sigma}_o = 0.7$ cm

Work in progress: Integrate surface current observations

Current available global data sets: SAR derived radial surface velocities (RVL) [7]

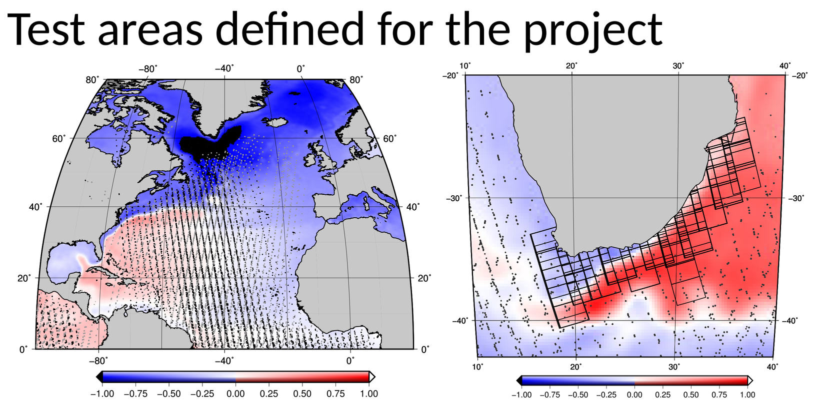
Idea within the PARASURV project

- motivated by Sentinel-1 OCN product
- C^1-smooth representation of DOT
- include directional derivatives as observations
- requires products to reduce ageostrophic currents

⇒ until now disappointing data quality, focus on methodology

⇒ potential SKIM mission promising

Test areas defined for the project



(a) North Atlantic Ocean (b) SW Indian Ocean

Available RVL observations (dots: wave mode (WV), boxes: interferometric wide swath mode (IW))

Summary and conclusions

DFG project to parameterize MDT with finite elements to integrate surface velocities to support separation of SSH observations

- implementation of C^1-smooth elements in test environment
- applied to model MDT signal
- directional derivatives can easily introduced as observations

⇒ prototype in test environment

- implementation of C^1-smooth elements in HPC environment
- integrate surface current measurements
- using SSH requires co-estimation of geoid, only possible with HPC

⇒ work in progress

Acknowledgments

The work is financially supported by the DFG project "PArametric determination of the dynamic ocean topography from geoid, altimetric sea surface heights and SAR derived RAdial SURface Velocities – PARASURV" (BR5470/1-1). The authors gratefully acknowledge the Gauss Centre for Supercomputing e.V. (www.gauss-centre.eu) for funding this project by providing computing time through the John von Neumann Institute for Computing (NIC) on the GCS Supercomputer JURECA/JUWELS at Jülich Supercomputing Centre (JSC).

References

[1] A. Albertella, R. Savcenko, W. Bosch, and R. Rummel. Dynamic ocean topography: The geodetic approach. Technical Report 27, Institut für Astronomische und Physikalische Geodäsie, Technische Universität München, München, 2008.

[2] J. H. Argyris, I. Fried, and D. W. Scharpf. The TUBA Family of Plate Elements for the Matrix Displacement Method. *The Aeronautical Journal*, 72(692): 701–709, 1968. doi: 10.1017/S000192400008489X.

[3] S. Becker. *Konsistente Kombination von Schwerefeld, Altimetrie und Hydrographischen Daten zur Modellierung der Dynamischen Ozeantopographie*. PhD thesis, IGG, University of Bonn, Germany, 2012. URL `http://nbn-resolving.de/urn:nbn:de:hbz:5n-29199`.

[4] S. Becker, J. M. Brockmann, and W.-D. Schuh. Mean dynamic topography estimates purely based on GOCE gravity field models and altimetry. *Geophysical Research Letters*, 41(6):2063–2069, March 2014. doi: 10.1002/2014GL059510.

[5] S. Becker, M. Losch, J. M. Brockmann, G. Freiwald, and W.-D. Schuh. A Tailored Computation of the Mean Dynamic Topography for a Consistent Integration into Ocean Circulation Models. *Surveys in Geophysics*, 35(6):1507–1525, November 2014. 1573-0956. doi: 10.1007/s10712-013-9272-9.

[6] R. J. Bingham, P. Knudsen, O. Andersen, and R. Pail. An initial estimate of the North Atlantic steady-state geostrophic circulation from GOCE. *Geophysical Research Letters*, 38(1):L01606, January 2011. doi: 10.1029/2010GL045633.

[7] J. A. Johannessen, B. Chapron, F. Collard, V. Kudryavtsev, A. Mouche, D. Akimov, and K.-F. Dagestad. Direct ocean surface velocity measurements from space: Improved quantitative interpretation of Envisat ASAR observations. *Geophysical Research Letters*, 35(22), 2008. doi: 10.1029/2008GL035709.

[8] S. Müller. COSIMO — Consistent combination of satellite and in-situ data to model the ocean's time variable dynamic topography. Final Report of COSIMO project, ESA's Support To Science Element, IGG, University of Bonn, Germany, 2014.

[9] C. Neyers. *Integration von Radialen SAR Doppler Ozeanoberflächengeschwindigkeitsmessungen in die Berechnung der Dynamischen Ozeantopographie*. M.Sc. Thesis, Universität Bonn, Germany, 2017.

[10] M.-H. Rio, S. Mulet, and N. Picot. Beyond GOCE for the ocean circulation estimate: Synergetic use of altimetry, gravimetry, and in situ data provides new insight into geostrophic and Ekman currents. *Geophysical Research Letters*, 41(24):2014GL061773, 2014. doi: 10.1002/2014GL061773.

igg

Institute of Geodesy and Geoinformation
Theoretical Geodesy Group
University of Bonn

Contact:
Christian Neyers
neyers@geod.uni-bonn.de

Nussallee 17
D-53115 Bonn
www.igg.uni-bonn.de/tg/

UNIVERSITÄT BONN