Adjustment of digital filters for decorrelation of GOCE SGG data

I. Krasbutter, J. M. Brockmann, B. Kargoll and W.-D. Schuh

Abstract GOCE satellite gravity gradiometry (SGG) data are strongly autocorrelated within the various tensor components. Consideration of these correlations in the least-squares adjustment for gravity field determination can be carried out by digital decorrelation filters. Due to the complexity of the correlation pattern the used decorrelation filters consist of a cascade of individual filters. In this contribution some of the properties of these filters and their application to GOCE SGG data decorrelation will be presented.

1 Introduction

The Tuning Machine, as part of ESA's High-Level Processing Facility (HPF), was designed with the purpose to obtain an independent gravity field solution by GOCE data (cf. [1, 7, 10]). The strong autocorrelation of GOCE satellite gravity gradiometry (SGG) data has to be considered within this gravity field determination to reach the ambitious mission goal of 1-2 cm geoid height accuracies at a resolution of at least 100 km. Due to the huge number of observations obtained throughout the GOCE mission (approx. 100 mio. observations per gravity gradient tensor component), the use of the full covariance matrix is not possible; to handle this problem [9] proposed to decorrelate the SGG data by digital filters.

The gradiometer noise has a complex autocorrelation pattern with sharp peaks and a strong increase below the measurement bandwidth (see Fig. 1). To take these individual sub-patterns into account, [11] suggested to use filter cascades to achieve a step-wise decorrelation of the SGG data. In this contribution we will present different individual filters (cf. Sect. 2), which can be arranged as various filter cascades.

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In Sect. 3 two appropriate decorrelation filter cascades for GOCE SGG data are introduced. This contribution ends with conclusions and an outlook.

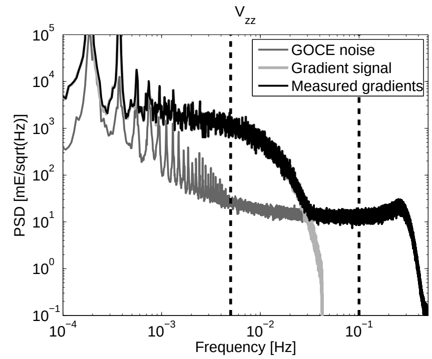

Fig. 1 Power spectral density (PSD) of the gradiometer noise (gray) for the V_{zz} component, the gradient signal (light gray), and the measured gravity gradients (black). The gradiometer noise has a complex spectrum with sharp peaks and a strong increase below the measurement bandwidth (frequency range between the dotted lines).

2 Individual filters for decorrelation

In practice, there exists a wide range of filters with distinct properties. For a decorrelation of GOCE SGG data three different types of filters are of special interest: (1) high-pass, (2) notch and (3) whitening filters. These three filters, which we will describe precisely in the next subsections, have the common property that they are linear, discrete-time filters and can be expressed by the autoregressive moving-average (ARMA) model equation

$$y_t = \sum_{k=1}^{p} \alpha_k y_{t-k} + \sum_{k=0}^{q} \beta_k u_{t-k}, \tag{1}$$

where u_t is the input data, y_t the output data, α_k, β_k the unknown filter coefficients, and p, q the filter orders.

2.1 High-pass filter

The general idea of a high-pass filter is to eliminate frequencies up to a specified cutoff frequency from the signal and maintain all the other (higher) frequencies. Many high-pass filters are used, which differ mainly in terms of filter order and width of the transition band. The transition band contains the frequencies, which are neither completely eliminated from the signal nor exactly maintained by filtering. Within the given context, the following three types of filters turned out to be most suitable: the difference filter, best-adapted polynomial filter, and Butterworth filter.

Difference filter: A difference filter determines the first differences between adjacent signal values. Its main advantage is its simplicity (i.e. $\beta_0 = 1, \beta_1 = -1$, all other coefficients are zero), its disadvantages are its relatively wide transition band and an amplification of high frequencies.

Polynomial filter: The idea is to fit a polynomial of degree N to $q+1$ equally spaced data points (filter input) and find the midpoint (filter output). The thusly obtained filter is a low-pass filter, which can be transformed easily into a high-pass filter (cf. [2, 4]). For instance, a high-pass filter fitted by a straight line through three data points results in: $\beta_0 = -\frac{1}{3}, \beta_1 = \frac{2}{3}, \beta_2 = -\frac{1}{3}$, all other coefficients are zero. This filter is only slightly more complex than the difference filter, but has the additional advantage of being a symmetric filter, hence its phase shift is zero.

Butterworth filter: Butterworth filters have the advantage that they can be designed by directly specifying the cutoff frequency and the width of the transition band (by specifying the number of poles of the transfer function, with an increasing number of poles resulting in a small width, cf. [6, 11]). Filter coefficients can be determined via the equations given in [11, Sect. 5.4].

2.2 Notch filter

Elimination of one particular frequency from the signal can be realized by a notch filter. The transfer function of such a filter, which is designed by specifying its poles and zeros, is zero for the frequency ω_0, which will be eliminated. The poles are utilized for minimizing the influence of the filter on the other frequencies (cf. [11, 4]). The order of this filter is always $p = 2, q = 2$ and the filter coefficients are defined by (cf. [12]):

$$\beta_0 = \beta_2 = 1, \quad \beta_1 = -2\cos(\omega_0), \tag{2}$$

$$\alpha_1 = 2(1-\delta)\cos(\omega_0), \quad \alpha_2 = -(1-2\delta), \tag{3}$$

where δ is the degree of impact of the filter on the neighboring frequencies of ω_0.

2.3 Whitening filter

In contrast to the filters described in 2.1 and 2.2, whitening filters affect and diminish all frequencies to some extent. This filter is used to level the remaining moderate autocorrelations over the entire frequency domain. These filters are determined via a data-adaptive least-squares estimation of the coefficients (cf. [3, 4]). In its most general form the whitening filter is described by an ARMA filter. In this case the estimation of α_k, β_k is based on a two-step least-squares adjustment. In the first step an MA filter, where $\alpha_k = 0$, for all $k \geq 1$ and typically $q = 1000$, is estimated; in the second step resulting residuals are used to determine the coefficients of a low-order ARMA filter, with typical filter orders $p = q = 50$ (cf. [11, Sect. 5.3]).

3 Filter cascade for correlated GOCE SGG data

As mentioned in Sect. 1 (see also Fig. 1) the GOCE SGG data displays a strong and complex correlation pattern. To achieve a full decorrelation, this must be completely reversed in the frequency domain. Due to its complexity, a filter cascade consisting of single filters with different characteristics are used. For this purpose, we found the following two filter cascades, comprising some of the filters described in Sect. 2, particularly suitable:

Cascade A: The first cascade consist of two filters: The first one is a difference filter for leveling the very highly correlated and inaccurate low-frequency part of the error spectrum (cf. Fig 1) and a whitening filter for the remaining frequencies. The inverse spectrum of this decorrelation filter is shown in Fig. 2, its disadvantage is that the sharp peaks of the error spectrum are not taken into account by such a design, its advantage is computational simplicity.

Cascade B: In comparison to cascade A, the sharp peaks are taken into account by a notch filter for each peak (cf. Fig. 2). Thus this cascade consists of a difference filter, several notch filters, and a whitening filter. This cascade design results in a more complex decorrelation filter with a high warm-up (i.e. loss of data caused by the fact that the filter produces invalid output values to be discarded as long as the filter does

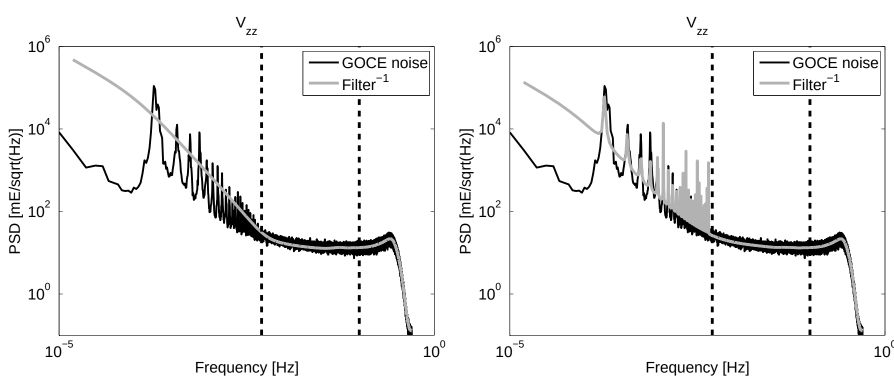

Fig. 2 Comparison of the gradiometer noise spectrum (black) to the decorrelation filter spectrum (light gray, inverse representation). Left: Filter cascade A, Right: Filter cascade B.

not have a complete set of given input values), which is a drawback especially for short data segments.

In both cascades the difference filter is used as high-pass filter and can also be replaced by a best-adapted polynomial or Butterworth filter. The result, especially the influence on gravity field solution, is in all cases the same as shown in [4].

4 Conclusions and outlook

Much effort has been put into the design of filter cascade to obtain an adequate decorrelation filter which is used in the gravity field determination. Both cascades described in Sect. 3 were applied and compared for decorrelation of GOCE SGG data (in [5] their different effects on the differences of the estimated coefficients to the ITG-Grace2010s model and on the estimated formal coefficient standard deviations were analyzed). Due to the simplicity and sufficient effectiveness of filter cascade A in the presence of data gaps (which occurred in large numbers in the past), this decorrelation model was applied for computing the three official GOCE gravity field time-wise solutions (cf. [8]). However, we anticipate filter cascade B to have superior performance in the case that future GOCE SGG data will have considerably less data gaps. Whether more sophisticated high-pass filters, such as best-adapted polynomial filters and Butterworth filters, will prove to be more appropriate than the currently used difference filter, will also depend on the future data characteristics.

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